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Analyzing of Slope Stability by Difference Model of Behavior

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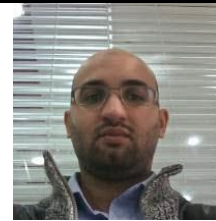
(Taleb Hosni Abderrahmane)

Abstract

Landslides are triggered around the world in different conditions, always are spectacular and often dramatic. The assessment and analysis of this problem is a very important act as risk prevention in geotechnical engineering. Many developments in geotechnical engineering research about applications, theory and practice to assess slope stability. All these improve the understanding of phenomena in realty. The researches of behavior of soils are also in development. There are numerous models of behavior of soils, from the elastic-plastic model to more sophisticated behavior models describing almost all aspects of elastic-visco-plastic behavior of soils. These models have been developed in order to be integrated into software. This article deals with the slope stability analysis, by the technique of shear strength reduction (SSR) coupled to the finite element method. Three criterions are used, the first is the Mohr–Coulomb failure criterion, and the second is the Hoek-Brown, the third is Drucker-Prager criterion. The factors of safety and the slide surfaces of this slope are analyzed by these criterions then compared and discussed. By making the comparative study, that can be captured and understood for appropriate use in engineering applications.

Keywords: Slope, Factor of safety, Slide surface, Criterions, Comparison.

Contents

1. Introduction

Landslides are triggered around the world in different conditions, always are spectacular and often dramatic, they are particularly encountered in large and important projects such as embankment, roads, dams, highways and tunnels, etc... The understanding of the mechanical processes that indicate or may indicate failure or even movements, that is firstly and necessary condition to assessment the stability of slopes. Vaunat, et al. [1] and Leroueil, et al. [2] they proposed a "geotechnical characterization of slope movements", for help to improve the understanding and analyzing slope movement, also for organizing the knowledge on slope behavior. Recently, Leroueil and Luciano [3] published paper of "assessment of slope stability" according to the recent developments about, the technologies for surveying and monitoring slopes; achievements about the understanding of slope behavior; and the development of procedures for numerical modeling of slope behavior. Analysis of slope stability is a challenging geotechnical problem. Often, different methods of analysis used to assess this problem will answer the same questions: is slope stable or not, safety factor, failure surface, type of failure, displacement. These questions contribute to the safe of this slope. The numerical modelling of slope stability is one of many procedures. It is starting by investigations insight and follows by measurements to detect the behaviour as a function of geological factors, climatic, hydrogeological, and others factors. When the slope formations are known, the slope stability analysis is an easy task for engineers by using computers. The results can help to understand these phenomena in complex physical processes. One of the most important things used in analyzing is the type of behaviour. There are various models of behaviour of soils, from elastic-plastic model to more sophisticated ones, that try to describe real models of behaviour of soils, and they have been developed in order to be integrated into software. Such as, Mohr–Coulomb, Drucker-Prager, Hoek-Brown, Cam-Clay, Tresca, and other. Many techniques exist to evaluation the stability of slopes, as Limit Equilibrium Method [4] widely used by researchers and engineers. The most common limit equilibrium techniques are the methods of slices [5]. In addition, numerical methods have been extensively used in the past several decades due to advances in computing power such as continuum methods [6] Finite Difference Method [7] Finite Element Method [6]. For discontinued methods, one quotes the Discontinuous Deformation Analysis [8] Discrete Element Method [8]. The basic purpose of slope stability analysis, is determining a factor of safety against a potential failure mechanism and then deducing the failure shape. The failure criterion mostly used in this assessment is the Mohr–Coulomb criterion. This paper focuses on the comparison between the Mohr–Coulomb (MC) and the Hoek-Brown (HB) and Drucker-Prager criterions to analysis the slope stability. we use in this study, the Finite Element Method coupled to the technique of shear strength reduction (SSR) and we compare between the factors of safety and shape of the slip surfaces.

2. Methods

Many techniques and methods to analyse slope stability have been developed. The limit equilibrium methods are most often used by researchers and engineers. The application of FEM in geotechnical analysis has become increasingly common, as computer performance has improved. In this section, we will give three methods briefly. We start by Limit Equilibrium Method, then Finite Element Method, finally, Limit Analysis Method.

2.1. Limit Equilibrium Method

For slope stability analysis, the (LEM) is widely used by researchers and engineers conducting slope stability analysis, because these are traditional and well established. The most common limit equilibrium techniques are methods of slices, such as the ordinary method of slices Fellenius [9] and the Bishop simplified, Spencer, and Morgenstern-Price methods. In these methods there are many differences among them about the slip surface or assumptions in force. We will give an example based on the shapes of slip surface assumed, the LEMs can be grouped in tow: the first group is methods of analysis which use circular slip surfaces include: Fellenius [9] and Bishop [10]. The second is methods of analysis which employ non-circular slip surfaces include: Morgenstern and Price [11]; Spencer [12]; and Sarma [13]; Janbu [14] and others. The slice methods have some common features and Zhu, et al. [15] have summarized them as follows:

- The surface of the sliding body is divided into a finite number of slices, this slices are usually vertical cut.
- The strength of the slip surface is mobilized to the same degree to bring the sliding body into a limit state. It means there is only a single factor of safety which is applied throughout the whole failure mass.
- The safety factor is calculated from force and/or moment equilibrium equations.

The definition of the Factor of Safety (FS) is the same for all these methods, factor of safety is commonly used to quantify the safety level of a slope [16] is defined as follows:

$$FS = \frac{\text{Shear strength of soil}}{\text{Shear stress } required \text{ for equilibrium}} \qquad (1)$$

The various slice methods of limit equilibrium analysis have been well surveyed and summarized in many studies such as Abramson, et al. [17]; Duncan [6].

2.2. Finite Element Method

Among the continuum methods, the Finite Element Method (FEM) is largely used to analyse the solid and structural mechanics [18-21]. The numerical methods, and in particular the finite element method (FEM), has developed rapidly and become increasingly popular for the slope stability analysis. Literature analysis of slope stability using FEM, based on the technique of shear strength reduction was reviewed by Duncan and Wright [4] and Griffiths and Lane [22] and by Li, et al. [23]. Generally, there are two approaches using the finite element method to analyse slope stability Rabie [24]. One approach is to increase the load of gravity and the second approach is to

reduce the strength characteristics. The second approach is adopted in this study using the finite element software. Generally, two major tasks coupled in the slope stability analysis: the computation of the factor of safety and the location of the critical slip surface. The definition of the factor of safety is not unique [25, 26]. The technique of strength reduction (SRM) is typically applied to calculate the factor of safety by progressively reducing or increasing the shear strength of the material to bring the slope to a state of limiting equilibrium [27]. In recent years, there have been various developments in the strength reduction method (SRM) for slope stability analysis. The technique is also adopted in several well-known commercial geotechnical finite element programs.

2.3. Limit Analysis Method

The limit theorems provide a simple and useful way of analyzing the stability of geotechnical structures. This method is a powerful mathematical tool that provides rigorous lower and upper bounds to the exact stability factor in slope stability problems. The soil is assumed to deform plastically according to the normality rule associated with the Coulomb yield condition. The applied of this method started by Drucker, et al. [28] and Drucker and Prager [29] to analysis slope stability undergoing plane strain failure, with rotational and translational failure mechanisms. The method of limit analysis is based on two theorems:

- The lower bound theorem, which states that any statically admissible stress field will provide a lower bound estimate of the true collapse;
- The upper bound theorem, which states that when the power dissipated by any kinematically admissible velocity field is equated with the power dissipated by the external loads, then the external loads are upper bounds on the true collapse load [29].

Currently, most the slope stability evaluations based on using the limit analysis are based on the upper bound method alone, such as Chen et al. [30, 31]; Donald and Chen [32]; Michalowski [31, 33] and Viratjandr and Michalowski [34]. We talk about LEM, FEM and LA methods, there are other methods to assessment the slopes, such as, the Discontinuous Deformation Analysis; Discrete Element Method, and many others, these methods having all of the advantages and disadvantages, none is perfect.

3. Criterions

There are many criterions, such as, Cam-Clay, Tresca, and others. Here we present three criterions briefly we start by Mohr–Coulomb, then Hoek-Brown, and finally Drucker-Prager Failure Criterion

3.1. Mohr–Coulomb Failure Criterion

The Mohr-Coulomb criterion is the most common failure criterion encountered in geotechnical engineering. Many geotechnical methods and programs require use of this strength model. The Mohr-Coulomb criterion describes a linear relationship between normal and shear stresses (or maximum and minimum principal stresses) at failure. The Mohr-Coulomb failure criterion can be written as the equation for the line that represents the failure envelope given by:

$$\tau = c + \sigma_n \tan \varphi \tag{2}$$

Where τ is shear stress; σ_n is normal stress; c is the cohesive strength, and φ is the internal friction angle. The failure criterion can be expressed in terms of the relationship between the principal stresses. From the geometry of the Mohr circle. The Mohr-Coulomb criterion for triaxial data is given by the following equation:

$$\sigma_1 = \frac{2c \cos \varphi}{1 - \sin \varphi} + \frac{1 + \sin \varphi}{1 - \sin \varphi} \sigma_3 \tag{3}$$

3.2. Hoek-Brown Criterion

Currently, the Hoek-Brown (HB) criterion [35] is one or the most broadly adopted failure criteria to estimate rock mass strength in rock engineering. Over the past many years, the HB criterion has been applied successfully to a wide range of intact and fractured rock types. The latest version is the Generalized Hoek-Brown (GHB) criterion presented by Hoek, et al. [36]. The equations are expressed as follows:

$$F = -(\sigma_1 - \sigma_3) - \sigma_{ci} \left(-m_b \frac{\sigma_3}{\sigma_{ci}} + s \right)^a \tag{4}$$

$$m_b = m_i \exp \left(\frac{GSI - 100}{28 - 14D} \right) \tag{5}$$

$$a = \frac{1}{2} + \frac{1}{6} \left(e^{\frac{-GSI}{15}} - e^{\frac{-20}{3}} \right) \tag{6}$$

$$s = \exp \left(\frac{GSI - 100}{9 - 3D} \right) \tag{7}$$

Where,

σ_1 and σ_3 are the major and minor principal stresses respectively, and GSI is Geological Strength Index, σ_{ci} is uniaxial compressive strength of the intact rock [kPa], m_b is a reduced value of the material constant m_i and is given by the equations m_i is intact rock parameter, D is Disturbance factor. In application to rock engineering projects, it should be noted that the Hoek–Brown criterion is suited only to homogeneous, isotropic massive rock with few discontinuities or a heavily jointed rock mass [36-41]. The Hoek-Brown criterion used in this study is the 2007 version of the Hoek-Brown criterion [41]. Comprehensive review of the literature of estimating shear strength of the

Hoek-Brown criterion can be found in Carranza-Torres [40]. However, as Brown [39] has noted, deriving exact analytical solutions for estimating the shear strength of a rock mass modelled using the GHB criterion has proven to be a challenging task due to the complexities associated with mathematical derivation.

3.2.1. Equivalent Mohr–Coulomb Parameters for Hoek-Brown Criterion

The most geotechnical design calculations are performed with using the Mohr-Coulomb criterion, it is often necessary to calculate equivalent rock mass friction angles and cohesive strengths from the Hoek-Brown parameters. Moreover, most practitioners have more experience, and therefore an intuitive feeling for the physical meanings of cohesion and friction, which is not the case. In terms of equivalencies, the parameter for m_b is related to the frictional strength of the rock mass, and s, which is a measure of how fractured the rock mass is, related to the rock mass cohesion. These are only descriptive relationships, however. Where Mohr-Coulomb parameters are required, the fitting of the linear Mohr-Coulomb envelope to the non-linear Hoek-Brown envelope results in the following equations for friction angle φ and cohesive strength c.

$$\varphi = arcsin\left[\frac{6am_b(s + m_b\sigma_{3n})^{a-1}}{2(1 + a)(2 + a) + 6am_b(s + m_b\sigma_{3n})^{a-1}}\right][°] \qquad (8)$$

$$C = \frac{\sigma_{ci}[(1 + 2a)s + (1 - a)m_b\sigma_{3n}](s + m_b\sigma_{3n})^{a-1}}{(1 + a)(2 + a)\sqrt{1 + \frac{6am_b(s+m_b\sigma_{3n})^{a-1}}{(1+a)(2+a)}}}[KPa] \qquad (9)$$

Where, $\sigma_{3n} = \sigma_{3,max}/\sigma_{ci}$. Note that the value of $\sigma_{3,max}$ represents the upper limit of confining stress over which the relationship between the Hoek-Brown and Mohr-Coulomb failure envelopes is considered. A best fit of the Mohr-Coulomb line in the interval between $-\sigma_t \leq -\sigma_3 \leq \sigma_{3,max}$ leads to the Mohr-Coulomb parameters (φ, c).

The shear strength parameters of the equivalent MC generalized can be calculated by locating the tangent of HB envelope with the specified normal stress σ_n, as showed in Figure. 1(a) the slope of the tangent to the HB failure envelope gives angle of friction φ and the cohesion c given by the intercept with the shear stress axis
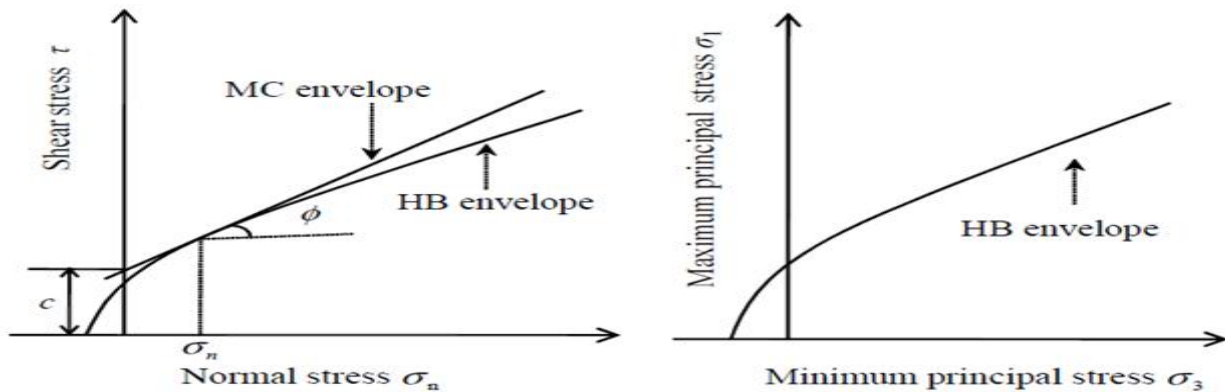

Figure-1. (a) Normal and shear stresses for the HB criterion, (b) Major and minor principal stresses for the HB criterion
Source: Jiayi [42]

3.3. Drucker-Prager Failure Criterion

The Drucker-Prager criterion was suggested by Drucker and Prager in 1952 Yingren, et al. [43] called the Extended Von Mises criterion as well, has been widely used for rock and soil materials and still attracts many researchers today [44] it is used to predict failure strength and be employed for plastic potential in continuum damage mechanic model. The advantages of the Drucker–Prager criterion are its simplicity and its smooth and, with the exception of some of the modified criteria, symmetric failure surface in the stress-space, which facilitate its implementation into numerical codes [45]. The Drucker-Prager yield function is given by:
Where

$$F = Mp + q - K \qquad (10)$$

$$p = \frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right) \qquad (11)$$

$$q = \sqrt{\frac{1}{2}\left(\sigma_x + \sigma_y\right)^2 + \frac{1}{2}\left(\sigma_y + \sigma_z\right)^2 + \frac{1}{2}(\sigma_z + \sigma_x)^2 + 3\tau_{xy}^2 + 3\tau_{yz}^2 + 3\tau_{zx}^2} \qquad (12)$$

The strength parameters of the Drucker-Prager model are, Friction coefficient M ; Cohesion k [kPa].

### 3.3.1. Equivalent Mohr–Coulomb Parameters for Drucker- Prager Criterion

The Drucker-Prager and Mohr-Coulomb surfaces can be matched in plane strain, (associated flow rule), as illustrated in Figure. 2 by the next parameters:

$$M = \frac{3\sin\varphi}{\sqrt{3 + \sin^2\varphi}} \qquad \begin{matrix}(13)\\(14)\end{matrix}$$

$$K = \frac{3c\ \cos\varphi}{\sqrt{3 + \sin^2\varphi}}$$

The equivalent k and M are given as function of φ and c in functions (13) and (14), where c and φ are the Mohr-Coulomb parameters. In analyses using an associated flow rule, the Drucker-Prager model with the equivalent Mohr-Coulomb parameters will produce results identical to the Mohr-Coulomb model. However, in elastoplastic calculations, the Drucker-Prager model will lead to a slightly less stiff response than the Mohr-Coulomb model.
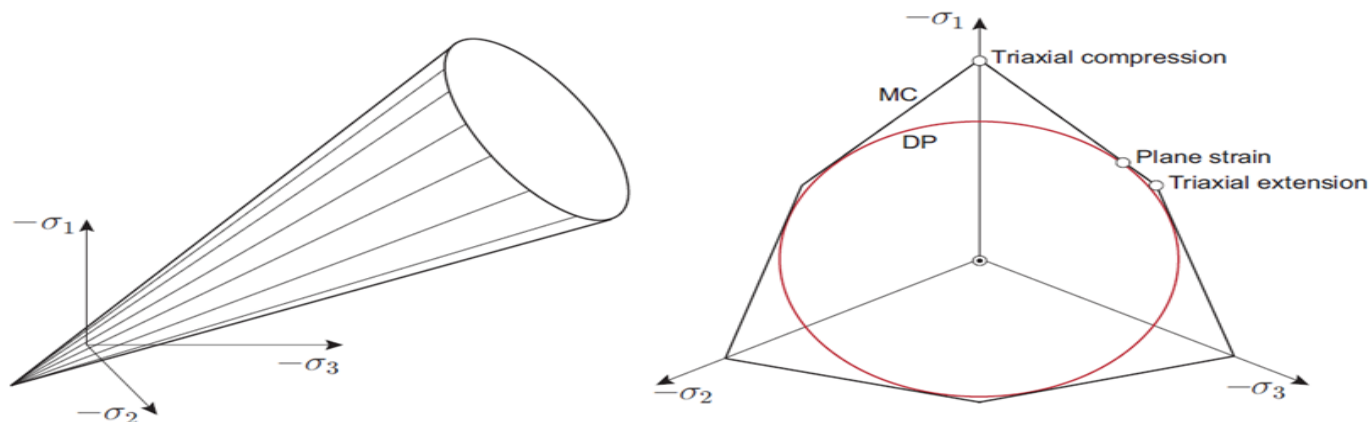
Figure-2. Drucker-Prager cone and corresponding to Mohr-coulomb criterion in plane strain "associated flow rule".
Source: Materials. Optum G2. Optum Computational Engineering. Pp 36-38. 2015.

4. Shear Strength Reduction

An important task in evaluation of soil slope stability is measuring or estimating the strengths of the slopes [46]. As we said previously, the strength reduction method (SRM) has been used to compute the factor of safety, and to trace the failure slip surface of a slopes, it is also called Phi-c reduction, this technique it is the most used in the programs of FEM and FDM to analysis slope stability. This method was used early in 1975 by Zienkiewicz, et al. [47] and has since been applied by Naylor [48]; Donald and Giam [49]; Matsui and San [50]; Ugai [51]; Song [52] and others. The main advantages of the SRM are as follows:

- The critical failure surface is found automatically from the application of the gravity loads and/or the reduction of shear strength;
- It requires no assumption on the inter-slice shear force distribution; and it is applicable to many complex conditions and can give information such as stresses, movements, and pore pressures.

4.1. Shear Strength Reduction by Mohr–Coulomb Failure Criterion

The Mohr Coulomb failure criterion, is the most used with the programs of FEM and FDM, for slope stability analysis, the SRM decrease gradually the strength parameters (c, φ) of the slope until instability occurs. The safety factor by SRM is the ratio between actual strength parameters and critical strength parameters, the corresponding formula is:

FS:

$$FS = \frac{c}{c_r} = \frac{\tan\varphi}{\tan\varphi_r} \qquad (15)$$

Safety of factor; c: Initial cohesive strength; φ: initial internal friction angle; c_r: reduced cohesive strength; and φ_r: reduced internal friction angle.

4.2. Shear Strength Reduction by Hoek-Brown Criterion

With Hoek-Brown criterion, the parameters reduced in technique of strength reduction analysis are, the uniaxial compressive strength of the intact rock (σ_{ci}); and the intact rock (m_i). The resulting factor is the strength based factor of safety:

$$FS = \frac{\sigma_{ci}}{(\sigma_{ci})_r} = \frac{m_i}{(m_i)_r} \qquad (16)$$

FS is safety factor; the σ_{ci} is the initial uniaxial compressive strength of the intact rock, and m_i is the initial intact rock parameter; $(\sigma_{ci})_r$ is reduced uniaxial compressive strength of the intact rock; and $(m_i)_r$ (m_i) is reduced intact rock parameter.

4.3. Shear Strength Reduction By Drucker –Prager Criterion

The parameters reduced in strength reduction analysis, with Drucker-Prager criterion are, k and M, in Figure. 3 is induce a state of collapse. The safety factor by the strength reduction analysis is given by:

$$FS = \frac{K}{K_r} = \frac{M}{M_r} \qquad (17)$$

FS is safety factor; the M is the Friction coefficient; M_r is reduced Friction coefficient; K is Cohesion; K_r is reduced Cohesion. The method used for the Drucker-Prager criterion is consistent with the one used for the Mohr-Coulomb criterion. As such, the tensile strength, k / M is unaffected by the reduction.
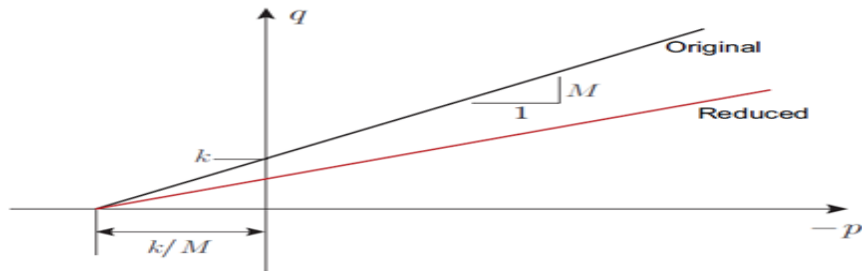
Figure-3. Original and reduced Drucker-Prager envelopes.
Source: Materials. Optum G2. Optum Computational Engineering. Pp 36-38. 2015.

5. Numerical Example

In this study, the Mohr-Coulomb, Hoek–Brown, and Drucker-Prager criterion, were used to the slope stability analysis, these failure criterions are commonly used to assess the strength slope stability. We consider the slope as shown in Figure 4. The results of our criterions are compared. Slope has a single layer, 14 m high and 33 m long, the Figure 4 gives all descriptions of this slope.
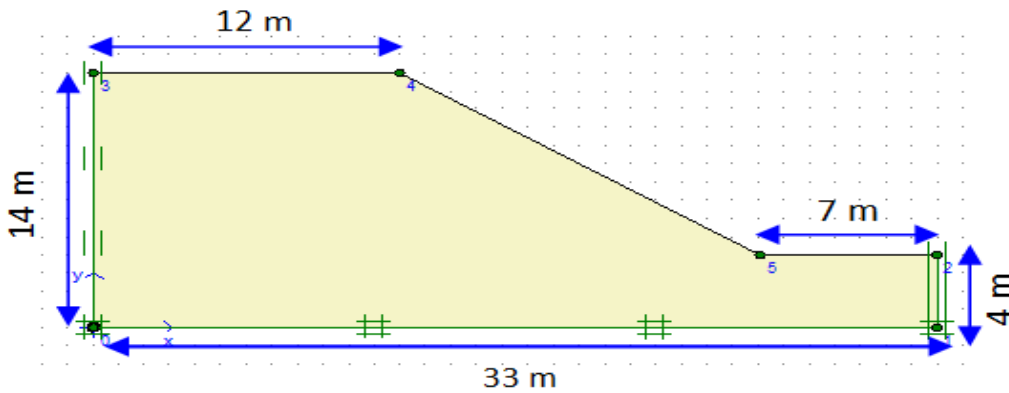

Figure-4. Slope geometry.

The physical and mechanical properties of the slope by our three criterions parameters are presented in the Table 1.

Table-1. Properties physical and mechanical of the slope

Material Type	Mohr-Coulomb	Hoek-Brown	Drucker-Prager	Units
E	30000	30000	30000	KPa
ν	0.25	0.25	0.25	-
c	7.8	-	-	KPa
φ	21.2	-	-	°
γ_{dry}	22	22	22	KN/m^3
γ_{sat}	22	22	22	KN/m^3
GSI	-	5	-	-
σ_{ci}	-	30000	-	KPa
m_i	-	2	-	-
D	-	0	-	-
K			12.33	KPa
M			0.613	

6. Results and Discussion

In this study, we investigate the slope with three different failure criterions, the first is equivalent Mohr-Coulomb, the second is the Hoek-Brown criterion, in last Drucker-Prager, and then we compare between their results (the safety factor, and the slope surface).The results are presented in the following Figures.
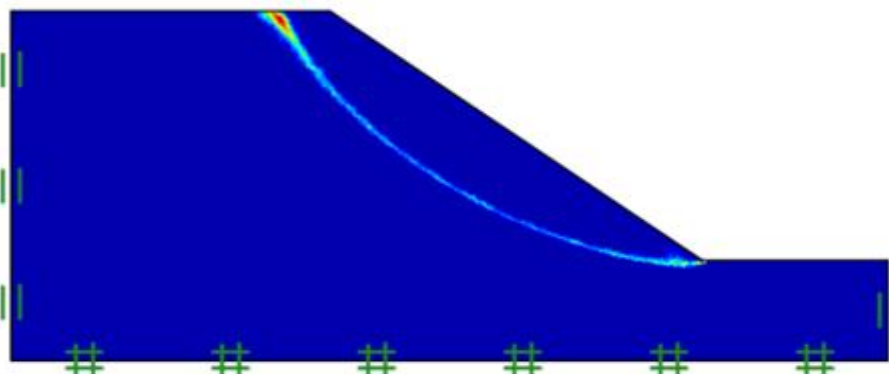

Figure-5. The failure surface by Mohr-Coulomb criterion

Figure 5 shows the failure surface in the slope analyzed by the equivalent Mohr-Coulomb criterion. The factor of safety value obtained from SSR analysis of this slope is FS=1.022, we noticed this slope is unstable.
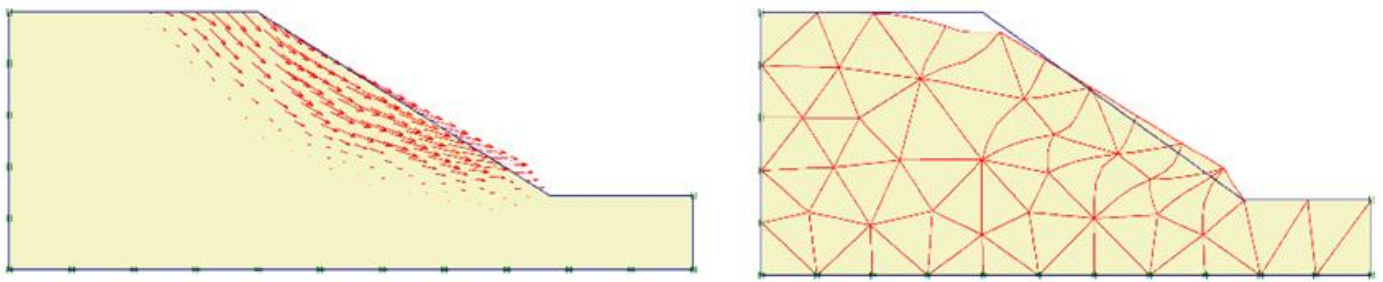
Figure-6. The displacement increments and mesh deformation by Mohr-Coulomb criterion.

In Figure 6 we have areas of displacement increments and mesh deformation of slope by the equivalent Mohr-Coulomb criterion, through strength reduction method calculation.
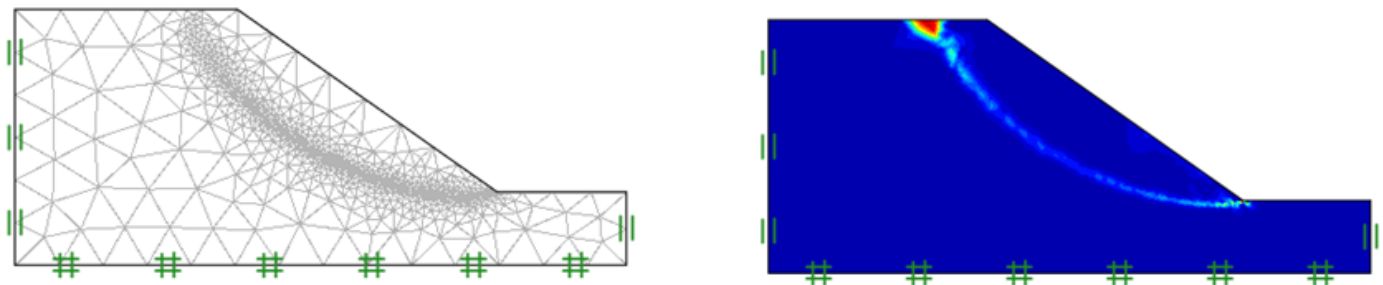

Figure-7. Failure surface by Hoek Brown criterion.

Figure 7 shows the mesh of failure surface of our slope, analyzed by Hoek-Brown criterion, the factor of safety value obtained by SSR analysis of this slope is FS = 1.766, the safety factor has indicated that the slope is stable.
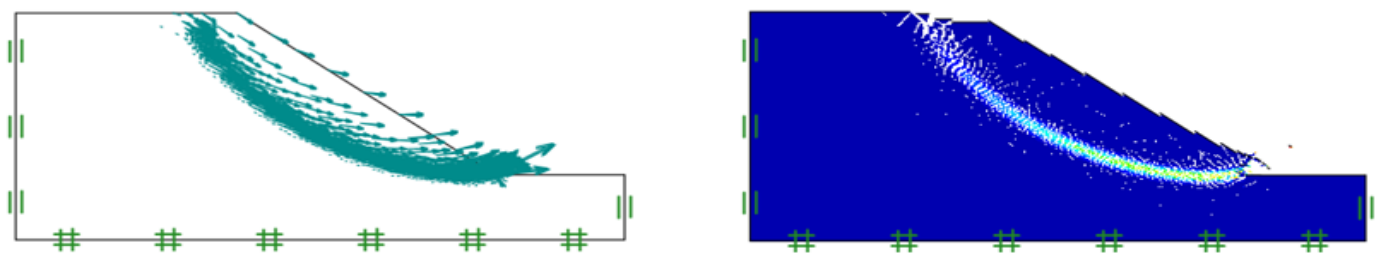

Figure-8. Displacement and deformation by Hoek–Brown criterion.

Figure 8 presents the shape of displacement and slope deformation for the Hoek-Brown criterion. The results obtained in this paper indicate many points; such as the factor of safety by Hoek-Brown criterion is upper than the factor of safety by Mohr-Coulomb criterion. In addition, the sliding surface of slope by the Hoek-Brown is larger than failure surface in the slope analyzed by the equivalent Mohr-Coulomb criterion.
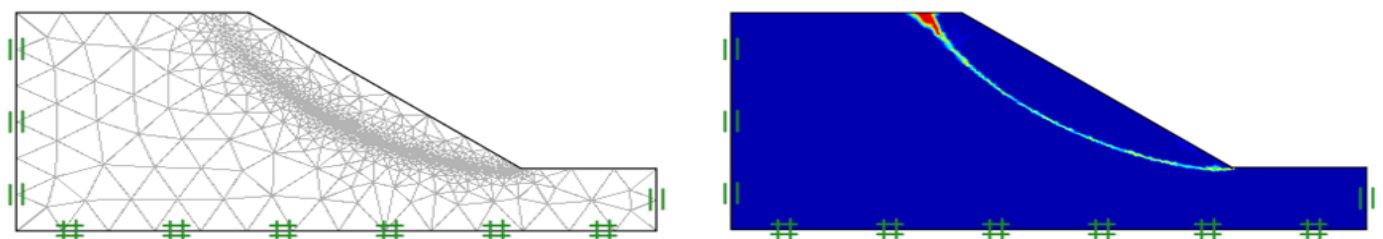
Figure-9. Failure surface by Drucker-Prager criterion.

Figure 9 shows the mesh of failure surface of our slope, analyzed by Drucker-Prager criterion equivalent of Mohr-Coulomb, the factor of safety value obtained by SSR analysis of this slope is FS = 1.018, we noticed this slope is unstable.

Figure-10. Displacement and deformation by Drucker-Prager criterion.

Figure 10 presents the shape of displacement for the Drucker-Prager criterion using the technique of shear strength reduction. The results of safety factor obtained in this paper by Mohr-Coulomb is similar to equivalent

Drucker-Prager, the safety factors have indicated that the slope is unstable. In addition, the sliding surface of our slope by Mohr-Coulomb criterion is the same as, surface in the slope analyzed by the equivalent Drucker-Prager criterion.
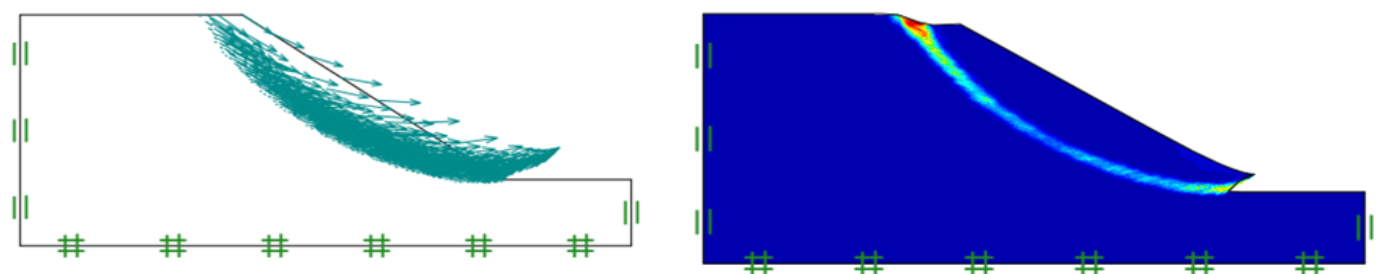
7. Conclusion

The behavior of soils play an important role in numerical analysis, there are numerous models of behavior of soils, from the elastic-plastic model to more sophisticated models that describe almost all aspects of elastic-visco-plastic behavior of soils, these models have been developed in order to be integrated into software. In this study, the discussion centers on the simulations slope stability by three criterions: Mohr-Coulomb, Hoek-Brown, and Drucker-Prager criteria. This paper has clearly shown many points, we found in our research, the powerful of the finite element method, and the advantage of shear strength reduction technique. We can analyze the slope stability by numerous criterions. In addition, the non-linear Hoek–Brown criterion is more complex than the Mohr-Coulomb and Drucker-Prager, it needs many parameters. The Mohr-Coulomb and Drucker-Prager models are widely used, owing to simplicity and easily determinable model parameters, in numerous engineering applications. We also can analyse the Mohr-Coulomb by equivalent parameters of Hoek–Brown criterion, and equivalent of Drucker-Prager. We can note also the results of Mohr-Coulomb model are similar to Drucker-Prager model, however the results of Hoek-Brown criterion are different. The results of safety factor changed by the change of criterions, that mean the influence of criterions in the strength reduction method, the shape of slope surface is the same but is not same biggest in these three criterions, that confirmed the strength reduction method is effected by these different models of behavior of soils.

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