Journal of Education and e-Learning Research
Vol. 11, No. 2, 332-344, 2024
ISSN(E) 2410-9991 / ISSN(P) 2518-0169
DOI: 10.20448/jeelr.v11i2.5565

Implementing blended learning to enhance the teaching of 3-dimensional trigonometry

Joseph Baidoo[1] ✉
Kakoma Luneta[2]

(✉ Corresponding Author)

[1]*Department of Maths, Science and Tech Education Walter Sisulu University Komani, South Africa.*
Email: jbaidoo@wsu.ac.za
[2]*University of Johannesburg, Faculty of Education, Johannesburg, South Africa.*
Email: kluneta@uj.ac.za

Abstract

This study investigates the impact of blended learning on the academic performance of grade 12 mathematics students with a focus on 3D trigonometry in comparison to traditional teaching methods. This research aims to understand the impact of technology on student outcomes influenced by the widespread use of technology in education. The study used an explanatory sequential mixed-method design to collect quantitative data from 381 grade 12 scripts and to obtain qualitative insights through interviews with 30 students and three mathematics teachers. This allowed for a more in-depth investigation of the effects of blended learning on student performance. According to statistical analysis, the results show a notable performance discrepancy with participants in blended learning outperforming their traditional teaching counterparts by an impressive mean difference of 19.2%. These findings align with prior research emphasizing advantages such as heightened teacher availability, flexible learning schedules and improved problem-solving skills associated with blended learning. The study underscores the crucial role of technology integration in education, particularly in enhancing conceptual understanding and addressing errors in complex topics like 3D trigonometry. It highlights the effectiveness of blended learning in elevating academic achievement in mathematics and recommends its integration into conventional teaching methods. Additionally, the study recognizes the value of digital equity and promotes more access to technology in schools as a means of reducing the digital gap and improving student performance.

Keywords: 3-Dimensional trigonometry, Blended learning method, MPCA error analysis, Traditional teaching method, WhatsApp, YouTube.

Citation | Baidoo, J., & Luneta, K. (2024). Implementing blended learning to enhance the teaching of 3-dimensional trigonometry. *Journal of Education and E-Learning Research, 11*(2), 332–344. 10.20448/jeelr.v11i2.5565

History:
Received: 24 July 2023
Revised: 2 February 2024
Accepted: 20 February 2024
Published: 15 April 2024

Publisher: Asian Online Journal Publishing Group

Funding: This study received no specific financial support.
Institutional Review Board Statement: The Ethical Committee of the University of Johannesburg, South Africa has granted approval for this study on 19 August 2021 (Ref. No. Sem 2-2021-044).
Transparency: The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.
Competing Interests: The authors declare that they have no competing interests.
Authors' Contributions: Both authors contributed equally to the conception and design of the study. Both authors have read and agreed to the published version of the manuscript.

Contents

Contribution of this paper to the literature
This study's originality lies in its meticulous comparison of blended and traditional teaching methods in 3D trigonometry revealing a statistically significant 19.2% performance difference in favour of blended learning. It emphasizes the role of technology tools like WhatsApp and YouTube in proposing practical recommendations for teachers to harness the benefits of blended approaches effectively.

1. Introduction

Teaching and mastering three-dimensional trigonometry or 3D trigonometry has always been challenging in the field of mathematics education (Luneta, 2015; Luneta & Makonye, 2010). Grade 12 learners often grapple with this complex mathematical concept leading to numerous errors and misconceptions (Makonye & Luneta, 2014). This study explores this persistent issue with the goal of illuminating potential solutions and pedagogical strategies.

A substantial body of literature has explored the challenges associated with teaching and learning 3D trigonometry (Ashlock, 2010; Luneta & Makonye, 2010). Previous research has highlighted the prevalence of errors and misconceptions among learners and identified the need for innovative instructional strategies to address this issue effectively (Baidoo, 2019; Luneta, 2015). Additionally, it has emphasized the growing role of blended teaching methods combining traditional face-to-face instruction with technology-based resources in modern education (Chowdhury, 2020; Ngu & Phan, 2023). This study poses a fundamental research question in light of this context: What are the significant differences in performance and exhibition of errors and misconceptions between grade 12 learners taught 3D trigonometry using blended teaching and those taught using traditional instructional methods? This question serves as the cornerstone of our investigation guiding us to explore the comparative impact of blended and traditional teaching approaches on learner performance, errors and misconceptions in the challenging domain of 3D trigonometry. This study seeks to change, the way this complex mathematical idea is taught and learned by providing insightful information about effective pedagogical approaches using a mixed-methods research methodology (Creswell, 2007; Huang, 2002).

2. Literature Review

The focus of sociocultural theory is on the role of culture in fostering collaborative discussions between students and more informed members of the community (Simon, 1995). Learners acquire the culture of the community (ways of thinking and behaving) through these interactions. Teaching strategies and materials for enabling learning are informed by teachers' conceptual frameworks on learning theories (Gehlbach, Mascio, & McIntyre, 2023). This is necessary because learners use semiotic, cultural and psychological tools to learn and manipulate activities (Huang, 2002).

It's essential to establish learning teams with defined responsibilities, identify actual-world situations and provide learners with support when using social views to help in the education of high school students (Gehlbach et al., 2023). Assignments, meaningful exercises and other activities that make sense in the context of the classroom should all be included in the interactive learning environments that teachers develop for their students. Learners could be grouped and a 3D trigonometry YouTube video could be shared on a WhatsApp group chat before a face-to-face classroom lesson. Teachers of mathematics in high schools must establish teams so that students may collaborate in order to create shared learning environments (Hadfield-Menell, Dragan, Abbeel, & Russell, 2016).

There are several ways in which technology can be used to promote the use of social learning theories in the classroom (Mutambara & Bayaga, 2020). When addressing social and real-world problems, using technology platforms will facilitate easier interaction and help us better comprehend all the variables (Rowe, Bozalek, & Frantz, 2013).Some users may find the vast variety of resources accessible intimidating but others may find it overwhelming. Huang (2002) and Simon (1995) posit that using technology to promote the use of social learning theories lets learners work together and learn 21st century skills. It's possible that some students and teachers may not be able to provide all the assistance required for online learning. This scaffolding should be reduced to a higher level platform so that all teachers and learners can use this technology more productively. Focusing on social learning in addition to technical learning might be helpful when examining how humans learn in a technologically sophisticated environment (Helsa, Darhim, Juandi, & Turmudi, 2021). Learners will also have greater access to problem-based learning that represents situated cognition, participation in a flipped classroom and a variety of interactions that facilitate robust and diverse communication central to the Vygotskian theory of instruction for in-person grouping through online environments and structured events.

2.1. Trigonometry

Trigonometry is an important part of mathematics at the high school level. Trigonometry is helpful when designing buildings, figuring out how to get around and making sounds for computers (Sholihah & Firdaus, 2023). In addition, trigonometry is used in calculating the speed and direction of the wind. In geology, structural geologists, seismologists and paleontologists also apply trigonometry in their computations (Ngu & Phan, 2023). Similarly, in criminology, trigonometry is useful when investigating a crime scene. The functions of trigonometry are vital in calculating the trajectory of a projectile and estimating the causes of a collision in a car accident (Ngu & Phan, 2023; Sholihah & Firdaus, 2023).

Previous studies revealed that students have preconceptions about comprehending mathematical subjects like trigonometry despite the fact that trigonometry is essential knowledge for many important functions, physics and construction (Asomah, Agyei, & Ntow, 2023; Ngu & Phan, 2023). The numerous misunderstandings shown in the learners' responses are frequently a sign of these challenges with high school mathematics (Egodawatte, 2011; Luneta & Makonye, 2010).

Ismail and Rahman (2017) admit that 3D shapes in secondary schools are considered a problematic topic. One of the key factors is that teachers and learners work in various stages (Van Hiele, 1959) and this influences their comprehension of critical concepts in 3D (Van De Walle, Karp, & Bay-Williams, 2019).

2.2. Traditional Method of Teaching Mathematics

The learners obtain an explanation of the concept or notion after the mathematics teacher is certain about the subject to be addressed in the traditional way of teaching mathematics. The teacher then tells the learners how to use a mathematical idea in a certain way to get to the correct answer (Sablan & Prudente, 2022). When a class activity is given, the traditional teacher guides the learners, telling them exactly how to use the materials or formulas in a prescribed manner. The prime focus of the lessons is to arrive at the correct answers. Learners emerge from these experiences with the view that the mathematics teacher is a knowledge possessor — a smart and intelligent source (Raja, 2018).

Furthermore, solving the paradigm or adhering to the norms of a computationally dominated, answer-oriented method of teaching will be unsuccessful and might create a culture that is a contributing reason for substandard performance in mathematics education (Naidoo & Singh-Pillay, 2020). According to Sreeshma, Samidass, and Rajkumar (2022), the conventional teaching approach encourages students to learn by memorization which prevents students from developing critical thinking and problem-solving abilities. In contrast, contemporary constructivists support the use of more interactive and learner-centered teaching methods. Recently, educationists have been advocating for blended learning also referred to as blended teaching (Capone, 2022; Fatimah & Herman, 2021; Helsa et al., 2021; Mhlanga, 2021).

2.3. Blended Learning

Modern technology makes it feasible to completely transform the manner in which knowledge is imparted to students. A blended teaching method which is a hybrid teaching approach describes a learning environment that combines teaching methods, delivery methods, media formats or a mixture of all these (McKenzie, Hains-Wesson, Bangay, & Bowtell, 2022). Usually, students can ask questions, participate in discussions, experience social contact and learn from others in a typical classroom setting. They can also have direct access to teachers. Some learners prefer an individualized or less structured environment (Herrera, Kavimandan, & Holmes, 2015). A self-paced learning resource such as a computer is needed by these learners (Helsa et al., 2021). It becomes crucial in these situations to use blended learning to break down the concepts. Effective blended teachers move skillfully across a range of learning modalities to personalize the learning of their learners. A mixed learning approach is recommended by Chowdhury (2020) due to the level of engagement and participation that learners experience which is also expected to foster conceptual development.

Chowdhury (2020) further submits that this approach allows learners to be responsible for the construction of their own knowledge. The merits of the blended teaching approach such as technological integration are what the National Curriculum Statement GR-12 promotes.

2.4. MPCA Error Analysis Protocol

Error analysis in mathematics identifies patterns of misunderstanding in learners' work to provide them with instruction according to their area of need. It functions to pinpoint and correctly identify the specific errors and associated misconceptions. The mathematical language error (M), procedural error (P), conceptual error and application (A) error protocols or MPCA error analysis protocol presented in Table 1 enlist prominent errors used for this study.

Table 1. MPCA error analysis protocol.

Error type	Indicators
Mathematical language error	An error associated with mathematical symbols or vocabulary.
Procedural error	Error as a result of a lack of understanding of the rules needed to simplify a problem.
Conceptual error	Error as a result of a lack of principles or properties for conceptual understanding underpinning the concept.
Application error	Errors are due to learners' inability to apply the known concept appropriately to a specific or different problem.

Source: Baidoo (2019).

3. Research Methods

3.1. Population and Sample

The target group for this study consisted of maths students in grade 12 in the Eastern Cape Province's Chris Hani West District.

The rationale for engaging learners in this grade was that they had appropriate exposure to 3D trigonometry concepts.

The Chris Hani West Educational District (CHWED) had 5610 grade 12 mathematics students distributed in 87 schools for the 2021–2022 academic year. Three hundred and eighty-one mathematics learners were sampled for this study. Six schools were chosen to participate in the research on the basis of two criteria: average mathematical achievement over three consecutive years (2018-2020) and the teaching methodology used for facilitating lessons. Schools with an average performance of 60% or higher for three consecutive years (2018-2020) were classified as high achievers; schools with an average performance of 30%-60% were classified as middle achievers and schools with an average performance of less than 30% were classified as low achievers. The mathematical performance of schools was calculated by averaging their mathematics performance over three consecutive years. The researchers also confirmed with the teachers at the selected schools to ascertain whether they were using conventional or blended teaching techniques. The sampled schools provided 381 scripts from the 2021 NSC. All of the province's schools contributed to the second trial mathematics paper for the National Senior Certificate.

3.2. Data Collection

Leaners scripts and focus group interviews were used as sources of data because they complemented each other and increased the validity and reliability of the data. The researchers realized that using learners' scripts would be advantageous since learners had been taught 3D trigonometry and the paper had been validated by Umalusi (the Examination Board of South Africa). The study used some of the scripts of the 2021 NSC Mathematics P2 trial learners specifically question 7. NSC 2021 Mathematics P2 question 7 focused mainly on three-dimensional trigonometry.

3.2.1. Focus Group Interviews

Focus group interviews were further used to elicit information. These are a type of in-depth conversation conducted in a group setting (Danielsen & Spanager, 2012). Meetings are characterized by the proposal, its size, make-up and interview methods (ibid). The purpose of this research was to further examine and comprehend learners' experiences with blended learning. The best way to achieve this was through focus group interviews. Interviews provided the researchers with the opportunity to dive deeply into a subject in order to uncover new information, highlight previously undiscovered facets of a problem and extract detailed, vivid descriptions based on the participant's own experience (Creswell, 2007). This study used a semi-structured interview method to identify potential misconceptions. Qualitative research circumvents the sample size constraint by allowing data saturation. Thirty learner-participants were interviewed. The focus group interviews were conducted perpetually until no new information or themes emerged from the learners' narratives. Table 2 presents a summary of the research study's data sources, sample sizes, types of collected data and the instruments and methods employed for data collection. This inclusive approach, incorporating both quantitative and qualitative data from diverse sources enables a detailed exploration of the influence of blended learning on 3D trigonometry performance, fostering a comprehensive grasp of the research inquiries. The data matrix below highlights the data collection sources.

Table 2. Data matrix.

Data source and sample size	Types of data	Instrument used for data collection	Data collection method
2021 NSC Trial mathematics Paper 2 Scripts n = 381	Students' responses to 3D trigonometry items only (Quantitative data)	Learners scripts	NSC 2021 trial maths P2 question paper
Grade 12 learners n =30 (In groups of 5)	Students' responses to 3D trigonometry and teaching methods items (Qualitative data)	Questionnaire	Focus group interview. Field notes

Analysis: Quantitative data from the learners' perspective

The data were analysed using a mixed analytic approach. Learners from different schools had their scripts assessed using two questions. The questions had a total score of 8 with the first question (Q7.1) having a possible highest mark of 5 and Q7.2 having a possible high of 3. A total performance score (a total mark out of 8 as a percentage) was computed and used in analyzing means across different categories. The figures summarize the characteristics of the schools from which scripts were drawn and the overall performance of learners in the two questions used for this study. These acronyms, FR, GA, HX, NT, QG and TA were used arbitrarily for the six schools selected for this study.

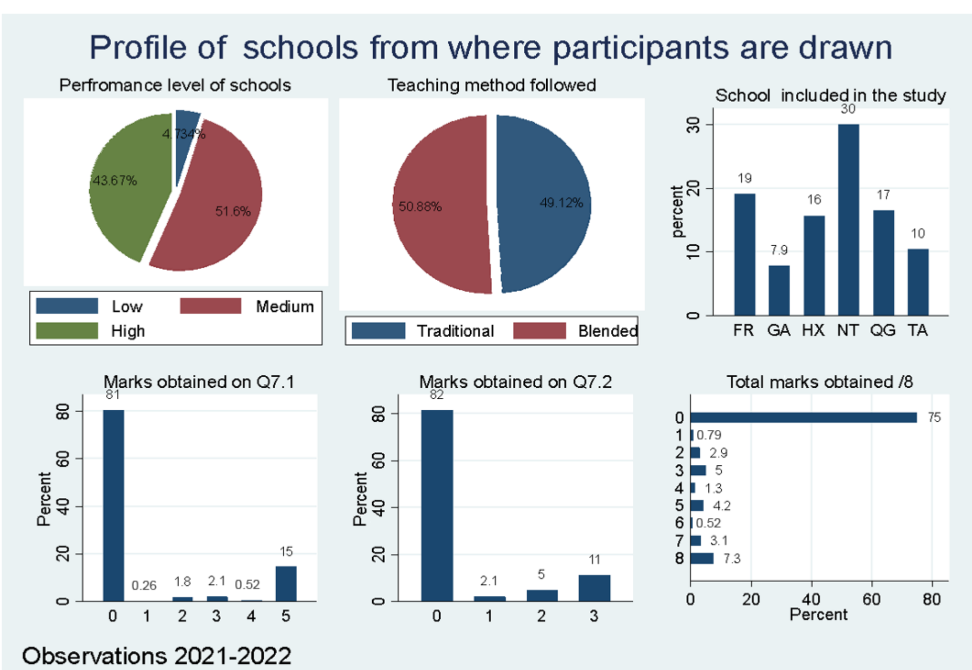


Figure 1. Profile of schools selected for the study.

Some of the descriptive characteristics in Figure 1 include the performance of the selected schools, the distribution of teaching strategies across the selected schools, the percentage of learner scripts employed and the performance of the students in the two 3-dimensional trigonometry problems under research. The researchers classified the schools into three groups, low, medium and high based on the information they had collected. Low performance was defined as being below 30% for three consecutive years, medium between 30% and 60% for three consecutive years and high as more than 60% for three consecutive years. 51.6% of the learner scripts used came from schools with average performance. However, the blended teaching approach received 50.88% and the traditional approach received 49.12%. It is suggested that a fair representation of the sample is essential for the conclusions drawn in this study as the main focus of the study was on using instructional methodologies to dissect 3-dimensional trigonometry issues.

Learners from different schools had their scripts assessed in relation to two questions. The total score was 8 with the first question (Q7.1) having a maximum possible score of 5 and the second question (Q7.2) having a maximum possible score of 3. A total performance score (a total mark out of 8 as a percentage) was calculated to analyse means across various categories.

Table 3. Distribution of schools across teaching methods and performance.

Recode of teaching	Performance			
	Low	Medium	High	Total
Traditional	0	188	63	251
	0.00	74.90	25.10	100.00
	0.00	86.24	51.22	65.88
Blended	40	30	60	130
	30.77	23.08	46.15	100.00
	100.00	13.76	48.78	34.12
Total	40	218	123	381
	10.50	57.22	32.28	100.00
	100.00	100.00	100.00	100.00
Pearson chi² = 129.19 Prob. = 0.0000				

The first row has *frequencies*; the second row has row percentages and the third row has column percentages.

A cross-tabulation was done to understand the distribution of schools across teaching methods and performance (see Table 3). 251 students (scripts) were taught using the conventional method; 188 of them (74.90%) attended medium-performing schools and 64 of them (25.10) attended high-performing schools.

However, there were 130 learners (scripts) of which 40 (30.77%) were from a low-performing school, 30 (23.08) from a medium-performing school and 60 (46.15) from a high-performing school. As a result, there are 40 (10.50%) low-performing schools, 218 (57.22) medium-performing schools and 123 (32.28%) high-performing schools.

3.3. Independent Samples T-Test for Means

Two methods of teaching are used in schools: traditional versus blended learning. Learners' performance in the two groups is compared using an independent sample t-test. The results show that 251 learners (scripts) were from the traditional teaching method and 130 learners (scripts) were from the blended approach with means of marks as a per cent (x/8 *100) of 9.9 and 29.1 per cent, respectively. The mean difference of 19.2% shows that those taught with the blended approach perform relatively better than those with a traditional approach. The difference is statistically significant which means it is not due to chance. It is a true reflection of the population from which this sample is drawn.

Table 4. Regression results.

Percent	Coef.	St. err.	t-value	p-value	[95% conf.	interval]	Sig.
School type: base~R	0	.	.	.	.	.	
GA	29.486	5.568	5.30	0	18.538	40.435	0.00
HX	47.611	4.474	10.64	0	38.814	56.408	0.00
NT	1.443	3.842	0.38	0.707	-6.112	8.998	0.00
QG	34.804	4.415	7.88	0	26.122	43.485	0.00
TA	-0.514	5.051	-0.10	0.919	-10.445	9.417	0.00
Constant	0.514	3.005	0.17	0.864	-5.395	6.422	0.00
Mean dependent variable		16.470	SD dependent variable		31.988		
R-squared		0.364	Number of obs.		381		
F-test		42.980	Prob > F		0.000		
Akaike crit. (AIC)		3560.243	Bayesian crit. (BIC)		3583.900		

Table 4 displays the results of a regression analysis examining the effect of school type on some outcome variable using the school FR as the baseline category. The regression coefficient (coef.) shows the estimated difference in the outcome variable for each school type compared to FR after controlling for other factors. For example, the coefficient of 29.486 for school GA indicates that GA schools scored 29.486% higher on average than FR schools, adjusted for other characteristics. The t-value and p-value show this is a statistically significant difference. Similarly, school HX and QG performed significantly better than FR. In contrast, the school types of NT and TA did not differ significantly from FR in their outcomes. The R-squared value suggests over a third (36.4%) of the variation in outcomes is explained by school type. The F-test and small p-value indicate school type is significantly related to outcomes.

Multiple regression was done to see the effects of the teaching method and prior school performance on trigonometry. The findings are shown in Table 5 which indicates that the percentage pass mark improves by 20% when blended learning is used instead of the traditional method. However, the average test score of students from medium-performing schools is 22% higher than that of students from low-performing schools. Individuals from high-achieving schools have markedly improved performance with an average difference in grades of 52% compared to those from low-performing institutions. R-square is 35.5% indicating that the two variables in the mode account for more than one-third of the variation in the percentage marks that students receive. This shows that the teaching method and current performance status of the school matter for learners' learning of trigonometry and 3D.

Table 5. Linear regression.

Percent mark	Coef.	St. err.	t-value	p-value	[95% conf. interval]		Sig.
Teaching method	-	-	-	-	-		
Base =Traditional blended performance	20.027	3.428	5.84	<0.000	26.768		0.00
1.Low	-	-	-	-	-	-	-
2.Medium	22.604	5.331	4.24	<0.000	12.121	33.087	0.00
3.High	51.823	5.012	10.34	<0.000	41.968	61.678	0.00
Constant	20.027	5.328	-3.76	<0.000	-30.503	-9.552	0.00
Mean dependent var		16.470	SD dependent var		31.988		
R- squared		0.355	Number of obs.		381		
F-test		69.179	Prob>F		0.000		
Akaike crit. (AIC)		3561.748	Bayesian crit (BIC)		3577.519		

Note: *** p< 0.01, **p< 0.05, * p< 0.1

The regression results in Table 5 reinforce the previous findings by identifying that the blended approach increased the pass mark by 20% juxtaposed to the traditional design. Regression analysis also outlined that medium-performing school learners performed better than low-performing ones. The findings ascertain that blended teaching methods influence higher mathematical performance among learners compared to traditional approaches. The results show that trigonometric performance was also impacted by an institution's performance level.

3.4. Qualitative Data: Learners' Perspective

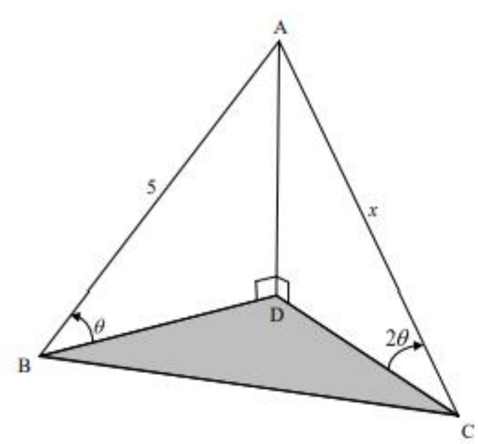
The data generated from the focus group was captured per school and categories were generated. The categories were then used to develop a theme. Learners were presented with four figures and asked to share with the researchers why they answered those questions the way they did:

Question paper

Mathematics/P2 10. SC/NSC DBE/September 2021(2)

QUESTION 7

In the diagram, B, C and D are in the same horizontal plane. AD is a vertical pole anchored by two cables, AB and AC. The angles of elevation from B and C to A, the top of the pole, are θ and 2θ respectively. AB = 5 units and AC = x units.



7.1 Show that $x = \frac{5}{2\cos\theta}$ (5)

7.2 Calculate the length of BC if it is given that $B\hat{A}C = 112°$ and $\theta = 30°$. (3)
[8]

Figure 2. Unitizing, categorizing and thermalizing: Learner responses.

Figure 2 illustrates a segment from the NSC Mathematics Paper 2 of 2020, featuring a 3-dimensional trigonometry question employed in this research.

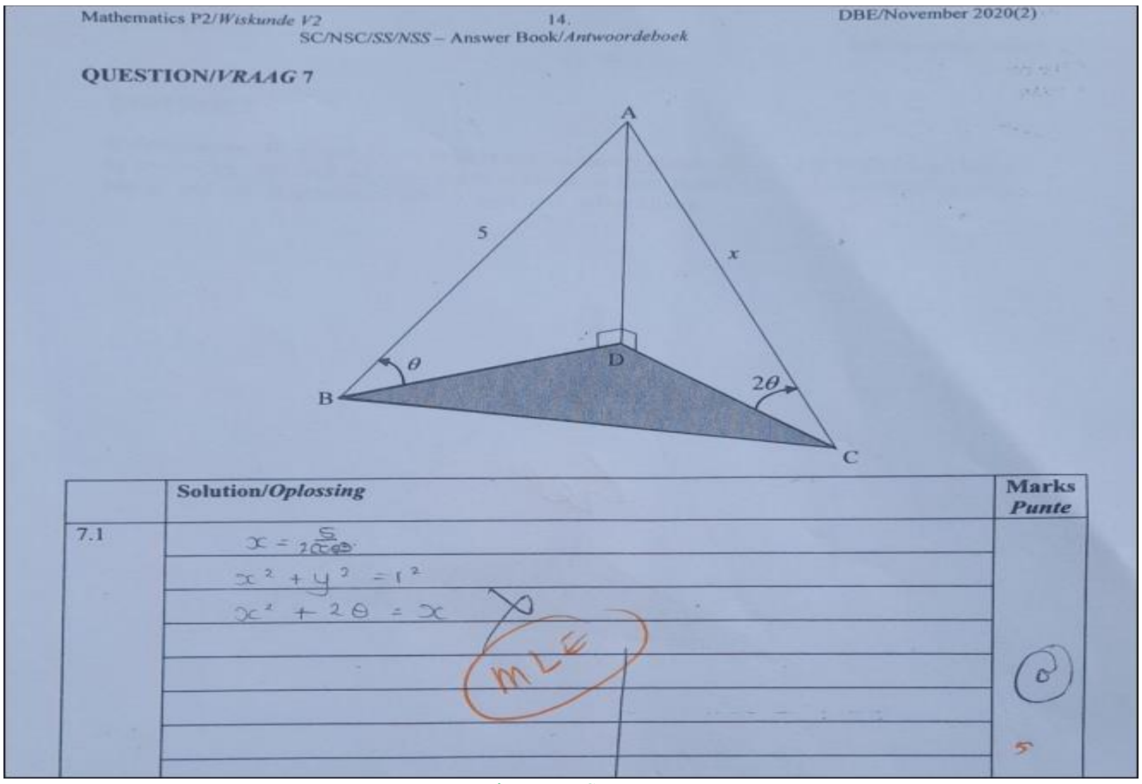


Figure 3. Scene 1.

Researcher: Look at scene 1 and share with me why the learner wrote that.

Learner NT (traditional and middle achiever): *The symbol theta was misuse[d] and the learner did not know when to apply Pythagoras' theorem.*

Learner HX (blended and high achiever): Pythagoras' theorem was used instead of the Sine rule.

Researcher: The learner swapped y^2 with 2 θ while trying to use Pythagoras, why?

Learner HX: When you are trying to solve for x you swap the variables.

Learner TA (traditional and low achiever): *learners were silent.*

Researcher: How did you solve this question? Can you please solve it for me?

Learner TA: (learners started to mumble) *It is difficult.*

Responses documented by the participating learners to scene 1 (see Figure 3) were linked to mathematical symbols and numerals as outlined by the MPCA error protocol. Consequently, codes such as the improper use of symbols and numerals were characterized as misuses of mathematical terminology and designated as mathematical language errors.

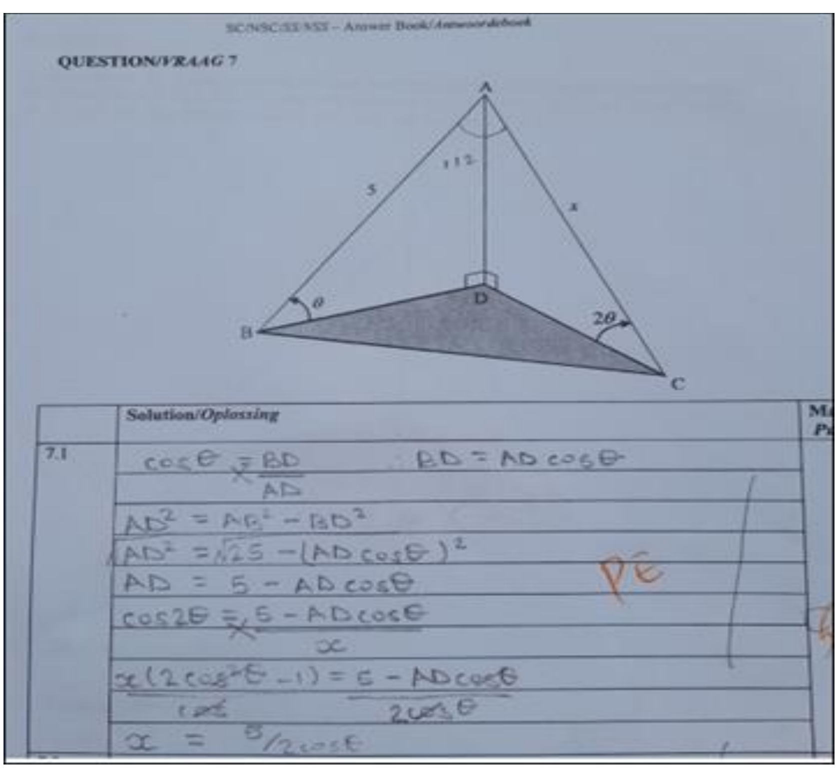


Figure 4: Scene 2.

Learner NT: *The learner failed to identify the sides of the* $\frac{BD}{AB}$ $for\ \cos\cos\theta$ *Instead, the learner wrote* $\frac{BD}{AD}$ for $\cos\cos\theta$. *The computation of the Pythagoras theorem was also wrong.*

Learner HX: The ratio of $\cos\cos\theta$ *wrong. Furthermore, the attempt to use sine was also inaccurate.*

Learner TA: I see Pythagoras $AD^2=AB^2-BD^2$. *I am not sure of the other steps.*

Respondents from NT and HX schools simultaneously identified computational errors in Figure 4. Mix-up steps and computational errors were coded as procedural challenges and themed as procedural errors. According to the MPCA error protocol, procedural errors are likely when learners' errors result from a lack of understanding of the rules required to simplify a problem.

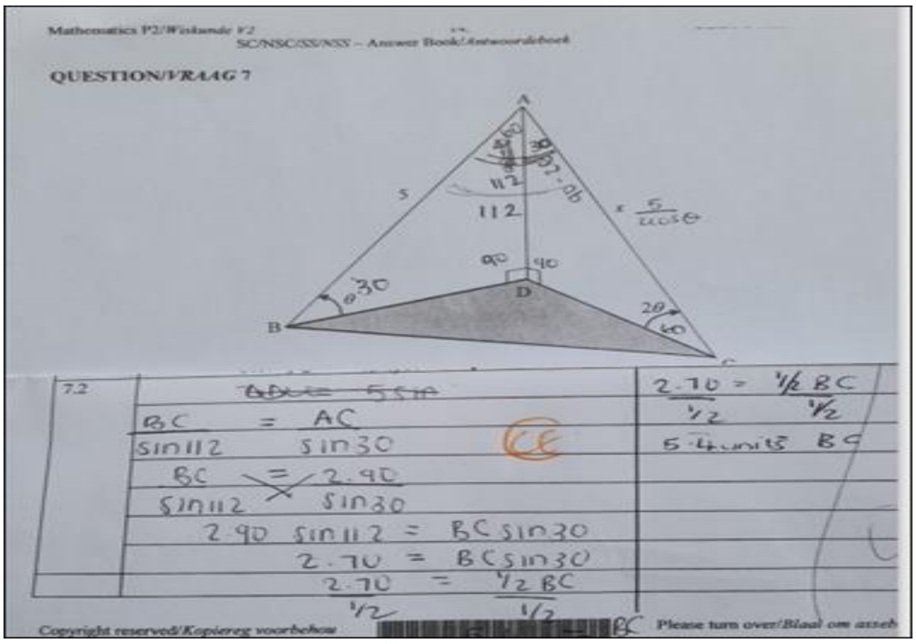


Figure 5. Scene 3.

Learner QG: The underlying planes of the 3-dimensional object were misunderstood, hence the learner's misuse of the faces of the planes.

Learner TA: The sine rule was used here.

Researcher: Are the steps correct?

Learner TA: Not sure

Learner HX: The ratios for the sine rule indicate the use of different planes. Secondly, if two sides (AB and AC) and an angle between the two sides are known then the cosine rule may be used.

Responses from learners in schools QG and HX to scene 3 (see Figure 5) were coded as misunderstanding the property of 3-dimensional planes and using the incorrect formula because of a lack of conceptual understanding. These codes were categorised as lacking a rational understanding of the concept. According to the MPCA error protocol, the conceptual error was the uncovered theme. According to the MPCA error categories, conceptual errors occur when students fail to understand the fundamental ideas or characteristics of conceptual comprehension.

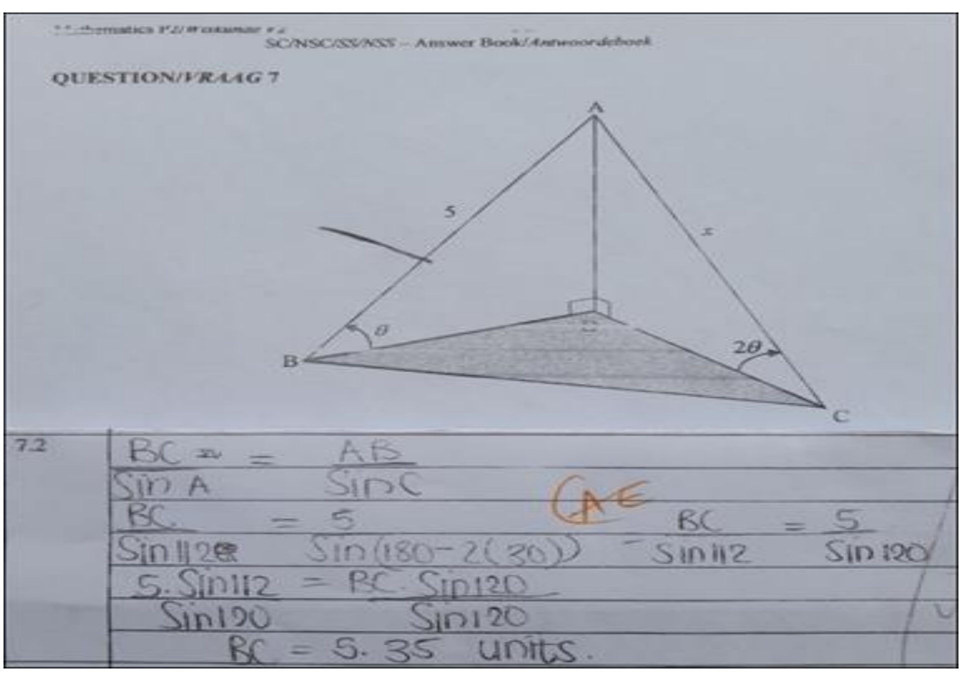


Figure 6. Scene 4.

Researcher: Look at scene 4 (see Figure 6) and share with me, why the learner wrote that. Is the answer correct?

Learner GA: The learner tried to use the sine rule for the proof but wrongly used the rule and forged the answer.

Learner QG: The sine rule is correctly defined but wrongly applied here. The cosine rule is appropriate to solve for the length of BC.

Learner FR: The sine rule was used again (learners smile again).

In question 7.2, learners were asked to determine the length of BC given that <BAC was 112^0 and theta was 30^0. Learners' responses to scene 4 (see Figure 6) shows that they can use the formulas they have chosen but only when they do so correctly. According to MPCA error protocols, application errors result from a failure to properly apply a known concept to a particular or unrelated problem.

Learner respondents were then asked whether they had computer, tablet or phone access, whether at school or home. All learners from schools HX, NT, QG and GA responded positively whereas 40% of learners from TA and FR (low achievers) did not have access to phones, computers or tablets.

Researcher: If yes, do you have enough access to data or Wi-Fi data for learning purposes?

All the learners indicated that they did not have adequate access to data or Wi-Fi for learning. In addition, learners from schools TA and FR expressed difficulties with network connectivity. The researchers became aware of the connectivity issues because when they visited schools in TA and FR to collect data, they experienced connection issues because the schools were somewhat remote.

Researcher: Do you watch mathematics content on YouTube to supplement learning like 3D trigonometry?

Again, learners from schools QG, HX, NT and GA responded affirmatively whereas learners from schools TA and FR responded negatively. Learners from schools TA and FR struggled with internet connectivity. Schools that have internet access have an advantage. Learners reported watching mathematics-related videos on YouTube to supplement their learning particularly when they did not appear to comprehend the topic.

Researcher: Do you watch mathematics content on YouTube to supplement learning like 3D trigonometry?

According to learners from the school HX, their teacher posted mathematical content before or after class.

Researcher: Do you belong to the school mathematics WhatsApp group? And how often do you make use of the WhatsApp group forum to supplement learning of mathematics content like 3D trigonometry?

Respondents from schools HX and QG claimed to be members of a WhatsApp groups specifically one for mathematics where these materials were posted. Learners in schools HX and QG reported watching mathematics videos that their teachers posted for them weekly while learners in FR reported viewing mathematics videos on their WhatsApp platform sporadically throughout the month.

Researcher: How would you compare the use of WhatsApp group or YouTube mathematics content to supplement teaching and learning with conventional face-to-face teaching methods?

A learner from school GA using blended instruction stated, “Learning is enjoyable because our mathematics teacher sends us mathematics videos and teaches us the content in the classroom.” Intriguingly, a learner from a traditional teaching approach (NT) school stated that “a blended approach is the preferred option because the use of YouTube videos facilitates learning because we can practice more without always relying on mathematics teachers.” Instead of watching unnecessary content on our phones, we should engage in additional video-based activities to improve our skills and comprehension. Another traditional approach learner from TA school asserted that having a WhatsApp group to view YouTube mathematics content would support their regular lessons but they were experiencing internet connectivity issues in this area. According to their responses, learners preferred a blended teaching method even though many had limited Internet access.

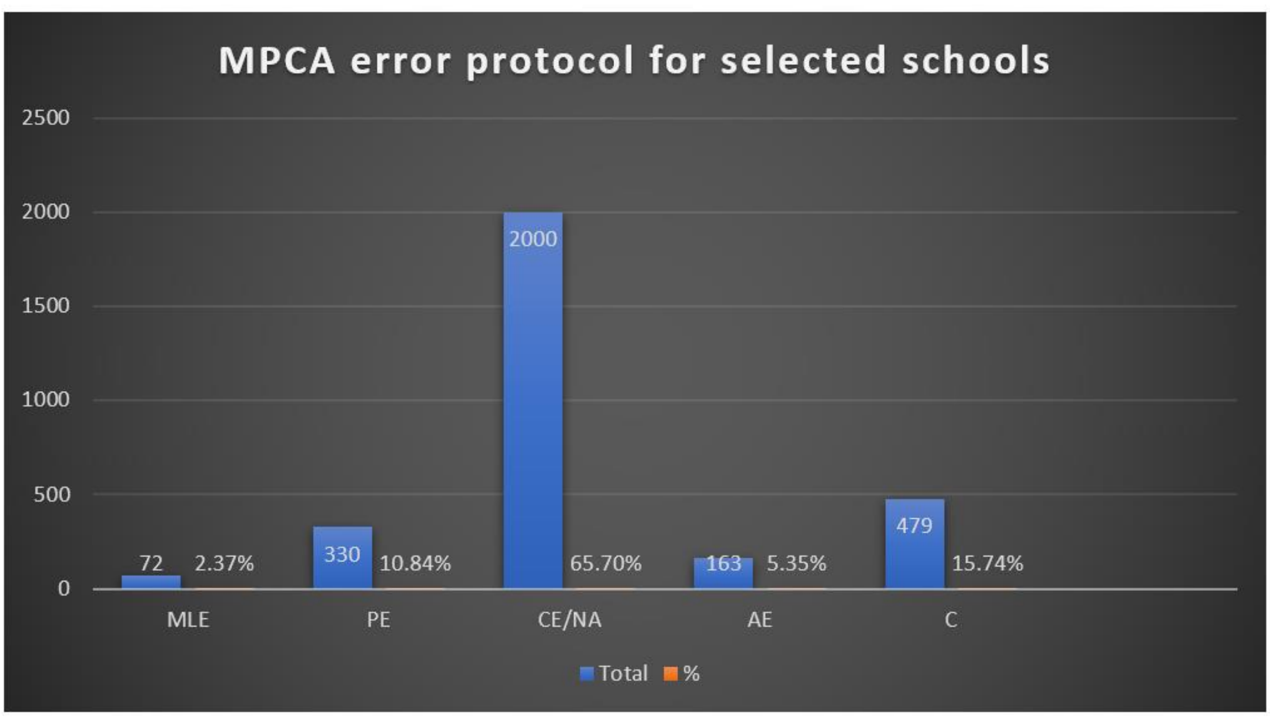


Figure 7. MPCA error protocol chart for selected schools.

The bar graph also shows the aggregate of MPCA error protocols for the errors that learners make when solving 3-D trigonometry problems (see Figure 7). Only 2.37% of the learners showed mathematical language errors while 65.80% showed conceptual errors, 10.84% showed procedural errors and 5.35% showed application errors. The analysis of the learners' work revealed that a lack of fundamental principles and properties of the three-dimensional trigonometry concepts might cause difficulties in solving 3-D trigonometry problems for high school learners.

4. Findings and Discussion

This paper compared the performance of grade 12 mathematics learners taught 3D trigonometry in blended learning and that of those taught in the traditional instructional method. The independent t-test analysis was used to compare learners' performance in the two teaching groups. The mean difference of 19.2% shows that those taught in a blended approach perform relatively better than those in the traditional approach. The difference reflects the population from which this sample was taken and is statistically significant not random variation. Multiple regression analysis also revealed that blended teaching increases the percentage pass mark by 20% compared to the traditional approach.

Previous research has demonstrated a number of advantages of blended learning that are linked to higher academic achievement which adds relevance to these findings. Teachers' availability and learning anytime and anywhere are the key benefits of learning through technological tools such as WhatsApp. WhatsApp enables learning beyond the classroom and the high availability of teachers to address learners' questions potentially improves the learning process. In addition, the use of technology in learning allows learners to build problem-solving skills on their own. Independent learners are more inclined to achieve higher academic standards (Chowdhury, 2020). Better learner performance in a blended learning group than in a traditional group may be attributed to regular contact between teachers and learners (ibid).

Teachers can also use YouTube videos and Google Classroom video resources to improve learners' understanding of concepts. These approaches provide authentic learning, problem-solving, critical thinking and collaboration that improve learners' performance. Consequently, Capone (2022) and Utami (2018) encourage teachers to develop blended learning platforms to match content with the most appropriate technological platform. The researchers emphasised that teachers get immediate feedback and that they better understand learners' thinking. Learning in the classroom is more productive and efficient when traditional classrooms give way to blended learning environments (Rowe et al., 2013).

Consequently, learners have more control over their education. Blended learners outperform conventional learners in maths problems because of these advantages. Moreover, blended learning improves test scores because it ensures that teachers use most of the class time to perform high-cognitive activities and attain a high level of learning (Chowdhury, 2020). Teachers can use most of their time to help students solve difficulties by implementing blended learning.

The lower performance of learners in the traditional learning group than in the blended learning group may be linked to the disadvantages associated with traditional learning. Sablan and Prudente (2022) emphasise that traditional methodology can no longer fulfil the needs of a rapidly changing world characterized by information explosion. Specifically, the traditional teacher-centred approach is ineffective in preparing learners (Sablan & Prudente, 2022). Consequently, Mulenga and Marbàn (2020) suggest that traditional schooling should be replaced with blended teaching because of its higher reliability, accessibility and capacity to provide an immersive learning environment. Moreover, Bora and Ahmed (2020) emphasise that teaching and learning mathematics using blended technology makes the teaching process effective and enjoyable.

The better performance of learners in the blended learning group than those in the traditional learning group can also be linked to higher self-efficacy in the former. The current study revealed that blended learning empowers, gives a learner confidence and makes one more creative; one may make mistakes, but they are never shy to attempt. However, learners taught using traditional pedagogy lack these attributes. Lastly, it can be argued that learners who used the blended learning method had higher scores in 3D trigonometry because they accessed more content using technology. This is because they had gadgets and the internet to access additional content. Conversely, their counterparts in traditional teaching classrooms had limited access to the internet. Since they only accessed concepts provided by the teacher, they had less understanding of mathematical concepts and lower performance.

Learners taught using traditional learning methods were more likely to make conceptual and procedural errors than those who were taught using conceptual learning methods. As a result, it is advised that teachers should incorporate 4IR when they are instructing learners on the principles of 3D trigonometry. Learners' conceptual abilities can be improved through blended learning. When learners have a solid grasp of the fundamentals of trigonometry, they are far less likely to commit errors in their conceptual understanding.

In addition, the blended learning approach should be adopted since it encourages greater involvement and participation from learners leading to an improvement in their overall conceptualisation of course material. It can only be possible to lessen the number of procedural and conceptual errors committed in schools by addressing the gap in schools' access to technology tools and the internet. As a result, the government should address the gap by increasing the resources available in traditional schools in the areas of information and technology. In addition, teachers working in conventional schools want to receive training in applying 4IR in the classroom.

Reducing conceptual and procedural errors is one of the many benefits of using 4IR (such as WhatsApp) to teach mathematics to teachers. Higher-achieving students in blended learning groups view mathematical information on YouTube to supplement their education whereas students in traditional schools are not allowed to use technology. Learners who make use of 4IR will therefore make fewer mistakes in mathematics. Therefore, it is advised that conventional educational institutions spend more money on technology in order to promote students' interest in three-dimensional trigonometry. These technology tools such as YouTube and WhatsApp, correct errors and misconceptions by changing tedious and challenging topics into entertaining and interactive experiences. This is accomplished by transforming the contents.

Learners in the twelfth grade who were instructed in three-dimensional trigonometry through blended learning performed significantly better than their counterparts who received traditional training. Technology has made it possible for students to access material beyond the classroom. Teachers are also readily available to respond to questions raised by learners at the appropriate moment. In addition, the use of technology assists learners in independently developing their ability to solve problems. These advantages contribute to improved academic achievement in a variety of ways. As a result, it is strongly suggested that teachers implement integrated learning strategies when instructing learners in 3D trigonometry.

The primary objective of this paper was to investigate the impact of a common type of mistake and an educational strategy in the context of assisting high school learners in solving problems more efficiently using three-dimensional trigonometry. The findings revealed that learners in grade 12 are prone to making conceptual errors regularly. Teachers should use concrete models and a mixture of teaching approaches if they want learners to achieve higher levels of success in 3D trigonometry. Furthermore, they use blended instruction since they have found it to be more successful than the more conventional face-to-face method. In addition, successful instruction can be achieved if mathematics teachers have obtained the necessary mathematical knowledge and quality instructional methods for teaching mathematics. When teachers are aware of common misunderstandings, they can better guide learners towards knowledge of the fundamentals of 3D trigonometry. In a nutshell, teachers should consider using flipped classrooms and various digital networks (such as WhatsApp and YouTube) to provide mathematical films to their learners.

5. Recommendations

Insufficient competence in the aforementioned domains might result in failures that could impact students' academic achievements and also misuse limited resources. Actualization of the blended teaching method requires considering the teacher's mathematical knowledge and technological skills, the learners' capabilities and available resources. Teachers with inadequate knowledge and experience teaching mathematical concepts will need help implementing blended methods. In addition, teachers who have a sufficient understanding of the concepts troubling learners are needed to implement technological learning models. They can also create personalized learning materials to aid learners with comprehension problems.

They need adequate mathematical knowledge and quality instruction to implement the change (Hill et al., 2008). It is imperative for teachers to ensure that a larger number of students have access to educational resources through digital means. For example, if a teacher creates trigonometry videos and posts them on YouTube as additional learning materials, these must be accessible to all. Schools and teachers should select an appropriate blended teaching method that supports mathematical concepts. Teachers should adopt a blend of asynchronous and synchronous e-learning lessons to provide an alternative delivery method.

Some platforms may include Moodle, WhatsApp, Facebook, Google Drive or YouTube. Videos are critical facets of blended learning because they allow learners to learn concepts at their pace, counteracting the anxiety that may be present when solving problems in class. The approach involves using emerging technology to deliver educational instructions in real-time. As discussed earlier, two-dimensional shapes and figures in trigonometry classrooms can be boring. Using a synchronous class where teachers and learners engage in a recorded class using interactive boards can facilitate comprehension. The lessons can be accessed from any place, and they facilitate recording.

6. Conclusion

Learning trigonometry is an engaging and enthralling activity with various mathematical concepts and formulas. Conceptual and procedural misconceptions are the primary errors affecting learners in high schools. Blended classrooms or lessons can be offered through Zoom, Moodle or Google classrooms allowing learners to participate remotely. The blended approach can use emerging technologies to virtually provide learners with educational materials. Proper implementation of blended approaches in classrooms will improve mathematical comprehension and overall learner performance.

This article has compellingly demonstrated the efficacy of blended teaching methods in enhancing the performance of learners when it comes to tackling trigonometric problems. Moreover, it has been revealed that these methods play a pivotal role in minimizing the occurrence of mathematical errors which have long been identified as a significant obstacle to achieving high academic performance.

Trigonometry holds a substantial weight in national mathematics examinations, exerting a considerable influence on a learner's overall academic performance. Nonetheless, learners across the nation grapple with a fundamental lack of comprehension when it comes to trigonometry, as manifested through a plethora of errors encompassing conceptual misunderstandings, procedural errors, mathematical language errors and application errors.

Learning trigonometry can be a captivating journey, replete with diverse mathematical concepts and intricate formulas. Misconceptions about ideas and processes are the main causes of these difficulties as they are often the main obstacles that high school students face. The advent of blended classrooms facilitated through platforms such as Zoom, Moodle or Google Classrooms has opened up new avenues for remote learning. This blended approach leverages cutting-edge technologies to provide learners with a wealth of educational materials in a virtual setting.

The profound benefits of blended teaching were further underscored in the wake of the COVID-19 pandemic. During this unprecedented period, learners were compelled to adapt to new methods of study as schools remained closed. Digital platforms, including Zoom, YouTube and video lessons have become the lifeline for continuing education. Interestingly, the popularity of blended learning is still rising even with traditional classroom settings returning and there's no denying that it can improve student achievement in all subject areas.

When it comes to mathematics, the incorporation of trigonometry-related visualisations, simulations, and interactive games can significantly enhance students' comprehension beyond what can be achieved with traditional instruction resources. Assistive technology such as projectors and movies provides student with a three-dimensional perspective on trigonometric issues in contrast to traditional techniques that rely on books, pictures and chalkboards which only provide a two-dimensional understanding of topics.

In light of these compelling findings, it is recommended that educational institutions embrace blended teaching approaches as an integral part of their pedagogical arsenal. This will not only augment learners' understanding but also boost their overall academic performance, thereby paving the way for a more proficient and confident generation of mathematics learners.

7. Implications

This study carries substantial implications for educational policymakers particularly in the domains of curriculum development and teacher training. It underscores the value of integrating blended teaching methods, especially in subjects like mathematics into the national curriculum. Policymakers should consider devising and implementing effective strategies to equip teachers with the skills required to proficiently use these methods, potentially leading to improved overall learner performance. The findings emphasize the vital importance of adequately preparing mathematics teachers for the use of blended teaching techniques. Therefore, teacher training programs should incorporate modules on digital pedagogy to empower teachers with the competencies needed for leveraging technology to enhance learning outcomes. Furthermore, educational institutions should prioritize allocating resources towards the development and acquisition of digital learning materials and tools. Ensuring schools have access to the necessary technology will facilitate the successful implementation of blended learning approaches. Additionally, this study sheds light on the potential for increased learner engagement through blended teaching. Teachers are encouraged to explore interactive and enjoyable learning methods, which may lead to enhanced comprehension and retention of mathematical concepts. Lastly, this study opens avenues for future research into the specific impact of blended learning on various mathematical topics and diverse learner populations as well as the evolving role of technology in mathematics education.

8. Limitations

This study has certain limitations. Firstly, its scope is primarily confined to high school learners within a specific geographical region which may restrict the generalizability of the findings to other educational levels or diverse cultural and socio-economic contexts. Additionally, the study assumes equitable access to the requisite technology for blended learning among all learners, although real-world disparities in technology access could potentially affect the applicability of these methods for a broader learner population. This research primarily offers insights into the short-term impacts of blended teaching on performance and error reduction leaving the exploration of its long-term effects on learners' mathematical abilities and attitudes towards the subject for future investigation. Furthermore, the study does not extensively delve into the challenges and constraints faced by teachers in implementing blended learning leaving room for future research to illuminate teachers' perspectives and experiences. Lastly, external factors such as parental involvement and the home learning environment which may exert an influence on learner performance were not extensively considered in this study. Exploring these variables in future research endeavours could provide a more comprehensive understanding of the impact of blended learning.

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