Partial Shift Mapping Decoding Algorithm to PAPR Reduction in OFDM Systems

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Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is a kind of modulation technique which allows the transmission of high data rates over wideband radio channels subject to frequency selective fading by dividing it to several narrow band and flat fading channels. OFDM has high spectral efficiency and Robustness to multipath fading. In contrast high peak to average power ratio (PAPR) of the transmitted signals is a major drawback of multicarrier systems like OFDM. High PAPR causes the nonlinear distortion in the received data and reduces the efficiency of the high power amplifier in transmitter. To solve the problem many techniques such as SLM and PTS algorithms are proposed. Recently a new simple method with low complexity respected to the SLM and PTS as Partial Shift Mapping (PSM) is proposed by Xing et al. He showed that the PSM method can reduce the PAPR parameter respected the other mentioned methods, effectively. In this paper we will design the corresponding decoder to the PSM technique and will evaluate its robustness respected to the high power amplifier distortion and the AWGN channel. Simulation results will show that the PSM method has a better Power spectrum density and is less sensitive to the type of modulation and number of subcarriers.

Keywords: SLM, PTS, Partial shift mapping algorithm, PAPR reduction, Spectral density, PSM decoder.

1. Introduction

OFDM means Orthogonal Frequency Division Multiplexing and has a long history [1-6] which is mainly used in many wireless and wired applications [7-9], such as Digital Audio Broadcasting (DAB) [10], Digital Video Broadcasting-Terrestrial (DVB-T) [11], Asymmetric Digital Subscriber Line (ADSL) [12], WiMAX technology and high speed communication networks [13].

Recently, the OFDM technique is considered as a strong candidate for the fourth generation (4G) of mobile communication systems. OFDM is a multicarrier modulation which converts a high data rate streams into a number of lower data rate streams which are transmitted simultaneously over a number of narrow band flat channels using a well known subcarriers. In fact for reducing data rate we need to increase interval time of symbol duration and it would reduce inter symbol interference (ISI) phenomenon effectively which caused by multipath fading. By using a guard interval named cyclic prefix, ISI can almost completely be eliminated.

The main advantages of OFDM is its high bandwidth efficiency, robustness to multipath fading, its simplification of channel equalization and its low computational complexity based on using fast Fourier transform (FFT) method [4].

The main drawback of the OFDM is high Peak-to-Average-Power-Ratio. When the OFDM signal with high PAPR passes through a nonlinear power amplifier, the signal will suffer from a nonlinear distortion, significantly. Therefore it causes a severe distortion in both in-band and out-of-band. In order to reduce the signal distortion, it requires a linear amplifier which has a large dynamic range. However, this linear amplifier has poor power efficiency and also is so expensive [14].

The problem can be solved using a necessary and another alternative technique as PAPR reduction algorithms. The PAPR reduction method can be used as a sub-block in an OFDM system. Several methods are proposed for reducing the PAPR parameter such as selected mapping technique (SLM) [15, 16], partial transmit sequence methods...
(PTS) [17, 18], Clipping and filtering algorithms [19, 20], Tone Reservation (TR) and Tone Injection (TI) procedures [21], Active Constellation Extension (ACE) techniques [22, 23], block coding methods [24, 25] and nonlinear methods as clipping and companding [26-28].

PAPR reduction is achieved by the above mentioned methods at the expense of loss in data rate, increase in computational complexity, transmit signal power and bit error rate. For example in TR, TI, and ACE algorithms it is required a power level increasing in the transmit signal after the PAPR reduction. Side information is required in SLM and PTS that must be sent to the receiver. Therefore it will cause an extra power consumption and loss in data rate. The complexity is increased at both transmitter and receiver for the methods as coding, PTS, SLM, interleaving, TR, and TI. These methods have different PAPR reduction capability and computational complexities. It is important to consider the analysis of power saving in Power Amplifier due to PAPR reduction and power consumption of implementation due to computational complexity [29, 30].

In addition, in [31, 32] the pre-coding based techniques are described. The pre-coding based methods are constituted from a simple linear method without the need of any complex optimizations. In Baig, et al. [33] authors are proposed a Discrete-Cosine transform matrix (DCTM) based pre-coding algorithm in a random-interleaved OFDMA uplink system, also they combined the selected mapping (SLM) and DCTM pre-coded input data to reduce the PAPR parameter in the mobile WIMAX systems. They reduced the PAPR parameter by a hybrid method, effectively.

As proposed in Bäuml, et al. [34] Selected Mapping (SLM) technique is the most promising reduction technique to reduce the PAPR of OFDM system. However, the main disadvantage of SLM is high calculation complexity because the scheme requires the large number of IFFT computation sub-blocks. Various techniques are discussed in [35-37] which focused on the reduction of complexity in SLM method.

Recently In Ouyang, et al. [38], a new and low complexity method to PAPR reduction is proposed which named as Partial Shift Mapping (PSM). It is very similar to the SLM technique and utilizes the properties of the discrete Fourier transform therefore it needs only one IFFT sub-block and no need to any additional complex multiplication so it can be a good candidate to generate an OFDM signal with low peak power. In Ouyang, et al. [38] only the PAPR reduction algorithm and complexity analysis respected to the PTS and SLM methods is considered but the corresponding authors didn’t propose any algorithm to decode the data. So the PSM technique will need an algorithm to recovering the data in the receiver.

In this paper, we will design a decoding algorithm to the PSM technique in the frequency domain. Also the robustness of the algorithm will be evaluated respected to the distortion which can be caused by high power amplifier (HPA). In the last step the symbol error rate of the PSM method will be calculated in an AWGN channel. Also power spectral density of the PSM will be evaluated.

The paper is organized as follows: in section II the OFDM system and its corresponding parameters as PAPR parameter, CCDF and Rapp model for High Power Amplifier will be reviewed. In section III PSM technique will be reviewed, abstractly. In section IV the design of PSM decoder will be introduced and in section V the simulation results will be shown.

2. Overview of Ofdm Parameters

2.1. OFDM System

OFDM technique converts a frequency selective channel into a number of frequency flat sub channels by dividing the available spectrum into a number of overlapping and orthogonal narrowband sub channels where each of them sends own data using a subcarrier, independently. A block diagram of OFDM systems in baseband model is shown in figure (1).

![Fig.1. The OFDM System in Baseband Model](image)

At first, in transmitter the binary inputs are grouped to obtain a symbol in $M$-ary based. According to a predefined baseband modulation such as $QPSK$ and $MQAM$ with $M=16$, 64, 256 and so, the obtained symbols are modulated using a signal mapper subsystem. In the next step, an $S/P$ sub-block converts the serial input symbols to a data block as a vector form denoted by: $X=[x_0^1, x_1^1, \ldots, x_N^1]$. The size of data vector is ‘$N$’ which determine the number of subcarriers in OFDM signal. Any subcarriers will be modulated by the obtained symbols in data vector using IFFT algorithm and consequently, the time domain of the OFDM signal are calculated as the following equation:
Where \( L \) is an oversampled factor which can be set to any values as: 2, 4, 8 [21]. In this paper we will use \( L=4 \).

To prevent the effect of ISI in OFDM signals, a guard time which well known as cyclic prefix, must be add to it. The adding process is shown in figure (2).

![Fig-2. The OFDM Symbol after the Cyclic Prefix Addition](image)

Finally, the obtained OFDM signal is converted to serial and is transmitted to the receiver through a frequency selective channel which is often considered as a Rayleigh fading model with additive white Gaussian noise (AWGN). In the receiver after removing the cyclic prefix the obtained signal is demodulated and based on the estimated channel the data are recovered.

### 2.2. PAPR Parameter

The PAPR of a signal as \( x(n) \) is defined by the following equation which explains the ratio of peak power respected to the power average.

\[
PAPR = \frac{\text{max}_{n=0}^{N-1} |x(n)|}{E\{|x(n)|\}}
\]

(2)

Where \( E\{\cdot\} \) indicates the expectation of the signal. Usually, an oversampling factor as \( L=2, 4, 8 \) is often used to estimate the actual PAPR based on its discrete samples. To evaluate the PAPR performance accurately, from the statistical point of view, the complementary cumulative distribution function (CCDF) of the PAPR is used to describe the probability of large deviation event respected to the expected value as a given threshold such \( \text{PAPR}_0 \). CCDF function can be written as the following equation:

\[
\text{CCDF} = \text{Pr} (\text{PAPR} \geq \text{PAPR}_0)
\]

(3)

In multicarrier systems anyone can obtain a simple approximate expression about CCDF of PAPR parameter using Nyquist rate sampling theorem. From the central limit theorem, the real and imaginary parts of the time domain signal samples can be follow Gaussian distributions, each with a zero mean and a variance \( \sigma^2 \) when there are a large number of subcarriers. The equation (4) shows the related parameters.

\[
x(n) = x_r(n) + jx_i(n), \quad n = 0, 1, ..., N - 1
\]

\[
f_z(z) = f_z(x) = \frac{1}{2 \pi \sigma^2} e^{-\frac{z^2}{2 \sigma^2}}
\]

(4)

Therefore, the instantaneous power of the OFDM signal follows exponential distribution. The CDF of the amplitude of a signal sample is given by the following equations:

\[
|v(n)| = \sqrt{x_r^2 + x_i^2}, \quad n = 0, 1, ..., N - 1
\]

\[
f_z(z) = \frac{1}{\sigma} e^{-\frac{z}{\sigma}}, \quad z > 0
\]

(5)

Finally the CCDF can be written as the following equation if it is assumed that any samples in multicarrier signal to be independent and identically distributed.

\[
p(\text{PAPR} > \text{PAPR}_0) = 1 - p(\text{PAPR} \leq \text{PAPR}_0)
\]

\[
= 1 - \sum_{n=0}^{N-1} p(z(n) \leq \text{PAPR}_0) - \text{PAPR}_0)^n
\]

\[
= 1 - \sum_{n=0}^{N-1} \left(1 - e^{-\frac{\text{PAPR}_0}{\text{PAPR}_0}}\right)^n
\]

(6)

### 2.3. High Power Amplifier Effects in OFDM Systems

When a multicarrier signal passes through a nonlinear device as high-power amplifier (HPA) or a digital to analog converter (DAC) a higher peak in input signal generates out-of-band energy and in-band distortion. These effects can be degrading the system performance, severely. The nonlinear behavior of an HPA can be modeled as two characteristics: AM/AM and AM/PM. There are many methods which can be used to modeling the distortion caused by HPA as Saleh, Gorbani and Rapp models.
3. Overview of the PSM Technique

In this section the Partial Shift Mapping (PSM) algorithm which has been proposed the first time by Xiang et al. will be reviewed [38]. As reported in Ouyang, et al. [38], PSM technique is very similar to the SLM algorithm. However, there is no need to multiple IFFT sub-blocks. PSM can be implemented using only one IFFT sub-block which has been proposed based on delay shift property of discrete Fourier transform. Equation (9) shows the delay shift property.

\[ X_i \cdot e^{j \frac{2\pi kl}{LN}} \overset{\text{IFFT}_{LN}}{\rightarrow} x((n - 1)_{\text{LN}}) \]  

Where \( X_i \) is the DFT of any signal as \( x(n) \) and \( l \) is corresponding delay time, also \((\cdot)_{\text{LN}}\) denotes the circular shift in module \( LN \).

In PSM method in order to reducing the PAPR four different sequences with length \( LN \) as the following equation is used.

\[ f_i(k) = \frac{1}{4} \left[ 1 + (-1)^{k-i} + (j)^{k-i} + (-j)^{k-i} \right] \]  

Where \( i = 0, 1, 2, 3 \) and \( f_i(k) \) is the \( i^{th} \) sequences which is used to changing the phase of subcarriers in OFDM symbol. These sequences have an important property as \( \sum_{k} f_i(k) = 1 \) for any \( k = 0, 1, 2, \ldots, LN-1 \). Therefore any OFDM symbol can be divided into four disjoint sub-signals by these sequences in the frequency domain as follow:

\[ X_{i,k} = f_i(k)X_i \]  

Where \( X_{i,k} \) is the \( k^{th} \) sub-symbol in the \( i^{th} \) sequence, \( X_i \) is the sequence.

Based on equation (9), the time domain signals of each sub-signals, \( X_{i,k} \), can be calculated by combining the copies of \( x(n) \) with different circular shift as \( LN/4 \), \( LN/2 \), and \( 3 \times LN/4 \) and different phases without any complicated multiplication operation. These sub-signals in the time domain can be written as the following.

\[ x,(n) = \frac{1}{4} \left[ x(n) + (-j)^{k}x(n + LN/4)_{2\text{L}2} + (-1)^{k}x(n + LN/2)_{2\text{L}2} + (j)^{k}x(n + 3LN/4)_{2\text{L}2} \right] \]  

In PSM algorithm the \( m^{th} \) signal candidate to PAPR reduction are calculated as follows:
\[ s^i(n) = (x_1(n) + x_2((n - l_{m,i})_i M + x_3((n - l_{m,i})_i M) \]  

(13)

Where \( l_{m,i} \) is selected randomly between 1 to \( LN \) and \( i = 1, 2, 3 \) and \( m = 1, 2, \ldots, M-1 \) where \( M \) is the number of the signal candidates to PAPR reduction.

Finally, PAPR parameter is calculated based on theses \( M-1 \) obtained candidates and original signal then is transmitted the one which has the lowest PAPR.

The CCDF of PAPR parameter in PSM method is shown in figure (4) based on a 16QAM modulation with considering a number of subcarrier as \( N=128 \) and oversampling factor as \( L=4 \) with different circular shift values as \( M=8, 16, 64, 128 \). As shown in the figure, when the value of \( M \) is increased the PAPR parameter is decreased effectively, for example it is shown that increasing \( M \) from 16 to 64 the PAPR of output signal decreases from 7.7dB to 7dB when we have the threshold probability of \( 10^{-3} \). Also power spectral density in this method which computed by Welch Algorithm is shown in figure (5). It is seen that there is no changes in the power spectral density when the number of circular shift is changed. In other words, the PSD isn’t more sensitive respected to variations of \( M \) parameter.

4. Proposed Method: PSM Decoder Design

In this paper we design a procedure to decoding the transmitted PSM signals in the receiver side. Using the following equations we at first will obtain the analytic formula to recover the original data from the PSM signal. Discrete Fourier Transform of the transmitted PSM signal denoted by \( x_{m,i}(n) \) can be written as follows:

\[ X_{m,i} = X_{m,i} + X_{m,i} e^{j \frac{2\pi}{LN} i} + \]  

\[ X_{m,i} e^{-j \frac{2\pi}{LN} i} + X_{m,i} e^{j \frac{2\pi}{LN} i} \]  

(14)

Where \( X_{m,i} \) is the DFT transform of the \( x_{m,i}(n) \). It can be calculated as follows using the combination of the equations (10), (11).

\[ x_{m,i} = 0.25(X_{m,i} + (-j)e^{\frac{2\pi}{LN} i} X_{m,i} + \]  

\[ (-1)e^{\frac{2\pi}{LN} i} X_{m,i} + (j)e^{-\frac{2\pi}{LN} i} X_{m,i} \]  

(15)

Therefore we can obtain all of the sub-signals based on different sequences as the following equations:

\[ X_{1,i} = 0.25(1+ (-j)e^{\frac{2\pi}{LN} i})X_{m,i} \]  

\[ X_{2,i} = 0.25(1+ (-j)e^{\frac{2\pi}{LN} i})X_{m,i} \]  

\[ X_{3,i} = 0.25((-j)(-j) - (-1) - j(-j))X_{m,i} \]  

\[ X_{4,i} = 0.25(1+ (-j)e^{\frac{2\pi}{LN} i})X_{m,i} \]  

(16)

with some modifications of the above equations the following results can be obtained.
Combining the above equations with the equation (14) we can obtain an important equation which can be used to decoding the received PSM signal as follow:

\begin{align}
X_{n,4k} &= X_{4k} \\
X_{n,4k+1} &= e^{j \frac{2\pi}{L} (4k+1)} X_{4k+1} \\
X_{n,4k+2} &= e^{j \frac{2\pi}{L} (4k+2)} X_{4k+2} \\
X_{n,4k+3} &= e^{j \frac{2\pi}{L} (4k+3)} X_{4k+3}
\end{align}

(18)

And finally we can decode the original data as the following equation using the received signal denoted by

\begin{align}
X_{n,4k} &= 0 \\
X_{n,4k+1} &= 0 \\
X_{n,4k+2} &= 0 \\
X_{n,4k+3} &= 0
\end{align}

(17)

Using the obtained equation the proposed system in order to decoding the original data based on received PSM signal and corresponding phase rotation side information can be designed. The corresponding decoder is shown as the block diagram in figure (6).

5. Simulation Results

In this section we show some simulation results in order to evaluate the PSM technique performance respected to the symbol error rate when the signal passes through HPA and AWGN channel, also comparison with the conventional OFDM will be investigated. All the evaluations will be considered based on the proposed PSM decoder. Table 1 shows all of the used parameters in all simulations.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values of Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of Baseband Modulations</td>
<td>QPSK, 16QAM, 64QAM, 256QAM</td>
</tr>
<tr>
<td>Number of Subcarriers</td>
<td>N=64, 128, 256</td>
</tr>
<tr>
<td>Algorithms</td>
<td>PSM Method</td>
</tr>
<tr>
<td>Number of OFDM Symbols</td>
<td>1000 OFDM symbols=1000×(N=64)=64000 data symbol</td>
</tr>
<tr>
<td>Calculation of PSD</td>
<td>Welch Algorithm</td>
</tr>
<tr>
<td>Oversample factor</td>
<td>L=4</td>
</tr>
</tbody>
</table>

Figure (7) shows the PAPR reduction performance of the PSM method with different baseband modulation schemes as: QPSK, 16QAM, 64QAM and 256QAM and the number of subcarriers N=128 with circular shift size as M=16. It is shown that increasing the modulation order didn’t change any more on the PAPR. Also in all modulation types the PAPR has been reduced about to 7dB when the probability of the given threshold is 10⁻³.
In the next step, also we will be going to show the effect of subcarriers number in the obtained results. As shown in figure (8) we apply the PSM algorithm by using the several number of subcarriers as $N = 64, 128, 256$ and 512 subcarriers assuming the modulation type as 16QAM with circular shift size as $M = 32$. The obtained result for PAPR is plotted in the figure. It is apparent that when the number of subcarriers is doubled the PAPR parameter is increased about 0.5dB. It is seen in figure (9) that there is a considerable enhancement in the power spectral density in out-of-band when the number of subcarriers is increased.

One of the most important problems in an OFDM system are in-band and out-of-band distortions which can be created by nonlinear high power amplifier, therefore there are some bit error rate (BER) performance degradation and spectral outgrowth respectively. These distortions are related to the level of backoff parameter in a high power amplifier. Therefore, to investigate the BER Performance of OFDM system based on the PSM technique, the OFDM signal with the assumed values of circular shift parameters is transmitted through power amplifier with different input backoff levels. The applied power amplifier model in the simulation is Rapp’s model.

Symbol error rate performance of the proposed method will be considered when there is an AWGN channel and a nonlinear high power amplifier with the Rapp model. We used the power amplifier parameters as $IBO=0, 3, 6$dB, $V_{sat}=0.3$ and different values to the smoothing factor as $s=0.8, 1, 2$. After passing the OFDM symbol through the nonlinear amplifier it is sent to the receiver through an AWGN channel which its SNR can be vary from 0 to 25dB. The original data is recovered using the proposed decoding algorithm and then symbol error rate is calculated in the receiver side.

At first we consider the effect of various modulations type as QPSK, 16QAM and 64QAM with $N=256$ subcarriers and circular shift size as $M=32$. The obtained results with the mentioned assumptions are plotted in figure (10), it is noticeable that when the order of modulation is decreased the obtained results are remarkable. For example, in the case of QPSK modulation it is required approximately 5dB signal to noise ratio to obtain the symbol error rate.
probability as $10^{-4}$. Also it is noticeable that in all of the modulation type the error probability is not sensitive to the IBO variation. Also in the figure (11) the effect of smoothing factor in the symbol error rate is plotted where IBO is set to zero.

5. Conclusion

In this paper a decoding algorithm to the Partial Shift Mapping in the frequency domain based on filter banks is proposed. PSM technique is the recently proposed method which is very similar to the SLM or PTS schemes. However in PSM method we use only one IFFT sub-block and it is a very simple respected to the other similar methods. Simulation results showed that the PSM algorithm is robust and less sensitive respected to the variation of the order of modulations and the number of subcarriers. Also a few enhancements are obtained when the number of subcarriers is increased.

References


