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# A Methodological Note on the Construction of High Frequency Macroeconomic Series: Evidence from Tunisia

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## Abstract

This note aims to formulate and to apply a combined method of the Loss Quadratic Function of Denton and the Best Linear Unbiased Estimator of Chow-Lin to construct quarterly data for Tunisia. High frequency series for GDP and total investment are obtained from related series which exist in high (quarterly) and low (annual) frequencies for the period 1970-2013. Tunisia began publishing quarterly GDP only since 2001Q1. We use these series to compare our estimates to those published by the Tunisian National Institute of Statistic (INS). Results show that the combined method generates high quality quarterly series.

**Key words:** High frequency data, Loss quadratic function, Tunisian data.
**JEL Classification:** C82.



## Contents

## 1. Introduction

Applied macroeconomics, especially time series econometrics, are eager in data (data intensive method). The quality of estimation depends crucially on the series length because modern econometric techniques, such as vector auto-regression, consume many degrees of freedom in the estimation process. In developing countries, macroeconomic high frequency series are very scarce and usually unavailable. Credible high frequency data are often required for econometric modeling and forecasting. This kind of data is useful to establish classical business turning points, assess the impacts of various government policies and external shocks. For example, Ilzetzki and Vègh (2008) noted that in order to study rigorously the cyclicality of fiscal policy, it's highly recommended to use high frequency data (quarterly).

In general, disaggregation methods, combine mathematical and statistical approaches. Mathematical approach which has been developed by Denton (1971) differs from statistical approach in that simulated data are not assumed to follow a specified statistical time series model. Statistical modeling approach encompasses ARIMA model based methods proposed by Hillmer and Trabelsi (1987) and state space models proposed by Durbin and Quenneville (1997). The choice of a particular method depends to a large extend on the information available for estimation and subjective preference or operational criteria.

Depending on the availability and the quality of benchmarking indicators, we usually distinguish three categories of temporal disaggregation methods:

- Methods without indicators,
- Methods with benchmarking indicators,
- Extrapolation methods.

In this article we are interested in the second category of methods. These methods lie on the hypothesis that the information contained in the quarterly benchmarking indicator is conform to that contained in the annual series.

In developing countries and particularly in Tunisia, most of the macroeconomic series are in annual frequency. The construction of high frequency series has been taken in the literature by Chow and Lin (1971), Denton (1971) and Fernandez (1981), and has been used by many economic institutions such as INSEE (France) Bournay and Laroque (1979), Banque de France (1986) the OECD. To construct a quarterly data set for Tunisia we use a combined approach of the quadratic loss function (QLF) of Denton and the best linear unbiased estimator (BLUE) of Chow- Lin.

This approach can be summarized as follows: the construction of high frequency macroeconomic series (X) can be obtained from related series (Z) on which data are available in both high and low frequencies. The procedure is to estimate regression coefficients, using annual totals of the dependent (Y) and independent variables (Z), and then apply these coefficients to the high frequency series to obtain preliminary higher frequency estimates of the dependent variable. As these estimates would not add up to the observed annual totals of the dependent variables, they are adjusted following the approach of Denton (1971).

The rest of the paper proceeds as follows. In Section 2 we describe the combined QLF and BLUE method. In section 3 we study the proprieties of the series used. We set the unit root tests and the cointegration relationships. Section 4 applies the QLF-BLUE method to a set of macroeconomic variables in Tunisia. Section 5 concludes.

## 2. The Combined QLF and BLUE Method

Assume that the low frequency series to be distributed $Y$=$[y_1, y_2, \ldots . y_m]$ is annual series with k intra- annual time period (k=4 in the case of quarters). Let the observed high frequency series cover m years and consist of n = m×k values. These series are represented in matrix form by $Z = [Z_1, Z_2, \ldots, Z_q]$. Where Zi $= [z_{i1}, z_{i2}, \ldots \ldots z_{in}]'$ i=1, 2……q are column vectors. The problem is to construct a new vector $X = [x_1, x_2, \ldots \ldots, x_n]'$ that:

- Makes use of the information available from the $Z_i$'s , and
- Satisfies the condition that the k values of the new series within each year sum to the observed annual totals for that year.

Assume that series to be estimated X, satisfies the relationship:

$$X = Z\beta + \mu \qquad (1)$$

$\beta = [\beta_1, \beta_2, \ldots . \beta_q]$' is a vector of unknown coefficients and $\mu$ is a random vector such that $E(\mu) = 0$ and $E(\mu\mu')$ = V.

V is unknown and can't be estimated directly. In order to estimate V one has to specify the process followed by the disturbances $\mu$. Different forms have been taken in the literature. Chow and Lin assume that the quarterly errors follow an AR(1):$\mu_t = \rho\mu_{t-1} + \varepsilon_t$ with $E(\varepsilon) = 0$ *and* $E(\varepsilon_t^2) = \sigma^2$and$|\rho| < 1$. Fernandez (1981) supposed that $\mu_t$ is a random walk and Litterman (1983) formulated a Markov-random walk which is an ARIMA(1,1,0), $\mu_t = \mu_{t-1} + \varepsilon_t$ and $\varepsilon_t = \phi\varepsilon_{t-1} + \upsilon_t$. More recently Wei and Stram (1990) have adopted the general case ARIMA (p,q,d) but in practice they use one of the above three methods.

Then, the observed low frequency series, Y, satisfies the relationship:

$$Y = S'X = S'Z\beta + S'\mu \qquad (2)$$

S $= \begin{bmatrix} J & \cdots & 0 \\ \vdots & J & \vdots \\ O & \cdots & J \end{bmatrix}$ is an n×m block matrix and where $J$ is column vector in which each element is unity and 0 is a k-dimensional null column vector. $J = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ if we look to construct quarterly data.

Fernandez (1981) formulated the QLF method of Denton and the BLUE method of Chow-Lin as follows: we minimize a quadratic loss function in the difference between the series to be created X and a linear combination of high frequency series Z. Subject to the constraint $Y = S'X$.

The program can be written $\begin{cases} Min\ [(X - Z\beta)'A(X - Z\beta)] \\ S.t \\ Y = S'X \end{cases}$ (3)

A is a symmetric n×n non singular matrix such that: $A = D'D$ where

$$D_{(n,n)}=\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & \dots & \dots & -1 & 1 \end{bmatrix}$$

Is a square matrix which converts the values of X and Z to first differences. The solution for X and $\beta$ of the program (3) are:

$$\begin{cases} \hat{\beta} = [Z'S(S'(D'D)^{-1}S)^{-1}S'Z]^{-1}Z'S(S'(D'D)^{-1}S)^{-1}Y \\ \hat{X} = Z\hat{\beta} + (D'D)^{-1}\, S(S'(D'D)^{-1}S)^{-1}[Y - S'Z\hat{\beta}] \end{cases} \quad (4)$$

$\hat{\beta}$is the General Least Square (GLS) estimator by regressing the annual values $Z'S$ of high frequency series on the low frequency series Y.

$\hat{X}$ is the estimator of the high frequency for the dependent variable (Y) and consists of two components.

The first, Z$\hat{\beta}$ applies the estimated regressing coefficients $\hat{\beta}$ to the observed high frequency series of explanatory variables.

The second is an estimate of the vector$\mu_{(n,1)}$ of residuals obtained by distributing the annual residuals $[Y - S'Z\hat{\beta}]$ with the matrix $(D'D)^{-1}$ S$(S'(D'D)^{-1}S)^{-1}$.

This method of generating data avoids the introduction of artificial discontinuity between the last period of one year and the first period of the next. The step problem occurs when benchmark to indicator ratio changes dramatically from year, given that the indicator that is used in the distribution process grows at different rate from the benchmark.

## 3. Proprieties of Tunisian Data

### 3.1. Unit Roots Test

In order to avoid spurious regressions, we first study the proprieties of the series. In Tunisia, most of the macroeconomic series are integrated of order one. In particular, using the ADF test, we verify that GDP, total investment and all the related series are integrated of order one. In order to have stationary and serially uncorrelated series we transform the series considered by the study to first differences. Unit roots tests for annual data are presented in table 1and table two presents the same tests for quarterly related data.

**Table-1.** Unit root tests of annual series 1970-2013.

| Variable | ADF | |
|---|---|---|
| | Level | 1st differences |
| GDP | -2.283 | -4.523 |
| IPI | -2.541 | -4.416 |
| I | -2.073 | -5.092 |
| CE | -1.984 | -3.873 |
| ICE | -2.352 | -4.625 |

**Table-2.** Unit root tests of quarterly series 1970-2013.

| Variable | ADF | |
|---|---|---|
| | Level | 1st differences |
| IPI | -2.731 | -5.503 |
| CE | -2.014 | -4.356 |
| ICE | -2.276 | -4.082 |

Results of table 1 and table 2 show that all the variables have a unit root. ADF test shows that they are first difference stationary. We can then look for stationary combination between variables in level through co-integration relationships.

To generate quarterly data for nominal GDP[1] we use the industrial production index (IPI) as a related series for which annual and quarterly frequencies data are available. This is judicious because industrial production is an important share of the Tunisian domestic product and because there exist a co-integration relationship between the two variables.

The quarterly series of total investment (I) is constructed using two related series which are total credit to the economy (CE) and the importation of capital equipment (ICE). This relationship could hold in Tunisia where financial market is still not well developed. Most of the firms' activity is still financed by the banking system through short and long term credit. Also, Tunisia is not an industrialized economy and the totality of capital goods is imported. In order to corroborate this idea we verify that the three series are co-integrated using the lambda max and the trace tests of Johansen (1988).

[1] We use nominal series for two objectives. First, to allow the user of the generated series, to choose the adequate deflator according to his goals. Second, using nominal series gives the possibility to compare generated series to the quarterly- nominal series published by the INS since 2000.

### 3.2. Cointegration Relationships

In order to generate suitable quarterly data we verify that there exist co-integration relationships between the dependent variables and the related series at low frequency. In this objective we use the trace statistic and the maximum eigenvalue statistic. The trace statistic for the null hypothesis of r cointegrating relations is computed as:

$$LR_{tr}(r|k) = -T\sum_{i=r+1}^{k}\log(1-\lambda_i)$$

Where k is the number of endogenous variables and $\lambda_i$ is the largest $i^{th}$ eigenvalue. The maximum eigenvalue statistic which tests the null hypothesis of cointegrating relations against the alternative of no-cointegrating relations is computed as:

$$LR_{max}(r|r+1) = -T\log(1-\lambda_{r+1})$$

Results are reported in table 3. They indicate that the GDP and the Industrial Production Index (IPI) are cointegrated and there exist a long relationship between Investment (I) and the two related series, Credit to the Economy (CE) and the Importation of Capital Equipment (ICE).

**Table-3.** Johansen cointegration Tests

| Cointegration relationship | $LR_{tr}$ | | $LR_{max}$ | |
|---|---|---|---|---|
| | $r = 0$ | $r \leq 1$ | $r = 0$ | $r = 1$ |
| GDP, IPI | 32.65 | 12.65 | 32.65 | 11.87 |
| I, CE, ICE | 40.35 | 10.13 | 40.35 | 10.04 |
| Critical Value 5% | 29.68 | 15.41 | 29.68 | 14.07 |

LR test indicates one cointegration equation for each relationship. The null hypothesis of no-cointegration is rejected at 5% critical value. These cointegration relationships allow to use related series to generate high frequency series of the dependent variables, namely Gross Domestic Product and Investment.

## 4. Generating High Frequency Series for Tunisia

The procedure has been implemented using R© CRAN software package. In order to compare our quarterly generated series for GDP[2] to those published by the Tunisian National Institute of Statistic (GDP true) beginning at 2001, we use the MAPE criterion as follows: $MAPE = \frac{100}{T}\sum_{2001}^{2013}\left|\frac{GDPtrue-GDPgenerated}{GDPtrue}\right|$. The calculation gives a value of MAPE= 1.4%. This means that the method adopted generates high frequency data with an error equal to 1.4%. We can see also through the plot that the two series are almost similar. Little differences are certainly due to the lack of information contained in the related series.

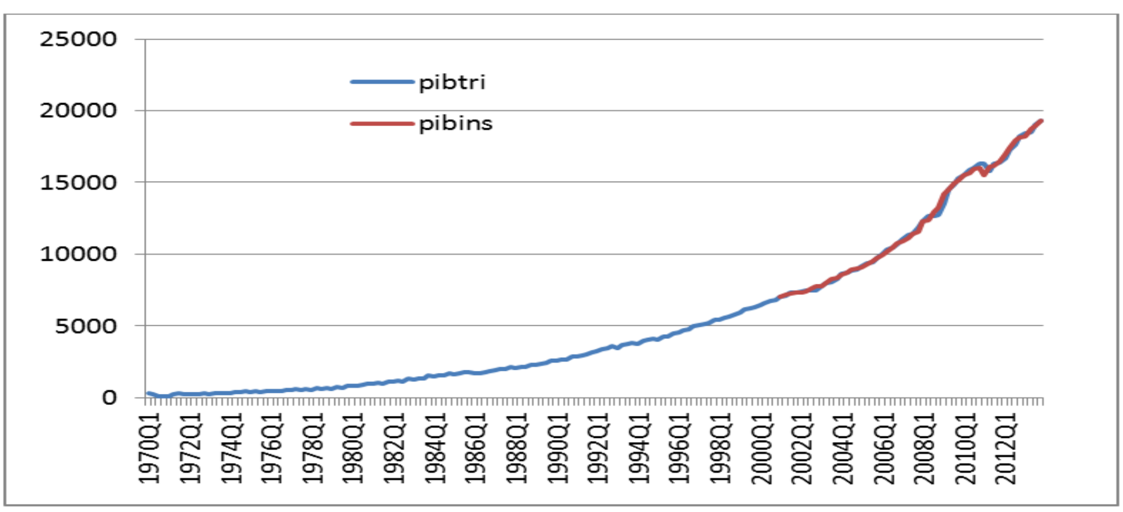


**Figure-1.** True GDP (pibins) and generated GDP (pibtri)

## 4. Conclusion

In this paper we have generated quarterly Tunisian data using a combined method of the Loss Quadratic Function of Denton and the Best Linear Unbiased Estimator of Chow-Lin. The generated series are useful for macroeconomic studies especially those which are interested with fluctuation and cyclicality. For example, most of the researchers agree nowadays that the study of the cyclicality of fiscal policy needs high frequency data.

## References

Banque de France, 1986. Méthode de trimestrialisation. Cahier Economiques et Monétaires, 23: 39 42.

Bournay, J. and G. Laroque, 1979. Réflexions sur la méthode d'élaboration des comptes trimestriels. Annales de l'INSEE, 36: 3-30.

Chow, G. and A.L. Lin, 1971. Best linear unbiased interpolation distribution and extrapolation of time series by related series. Review of Economic and Statistics, 53(4): 372-375.

Denton, F., 1971. Adjustment of monthly or quarterly series to annual totals: An approach based on quadratic minimization. JASA, 66(1971): 99-102.

Durbin, J. and B. Quenneville, 1997. Benchmarking by state space models. International Statistical Review, 65(1): 23–48.

Fernandez, R., 1981. A methodological note on the estimation of time series. Review of Economics and Statistics, 63: 471-476.

[2]The generated quarterly series for nominal GDP and total investment (I) in millions of Tunisian Dinars on the period 1970-2013 are available from the authors upon request.

Hillmer, S. and A. Trabelsi, 1987. Benchmarking of economic time series. Journal of the American Statistical Association, 82(December): 1064-1071.

Ilzetzki, E. and C.A. Vègh, 2008. Procyclical fiscal policy in developing countries: Truth or fiction. NBER Working Paper WP No. 10780, September.

Johansen, S., 1988. Statistical analysis of cointegration vector. Journal of Economic Dynamic and Control, 12(2-3): 231-254.

Litterman, R.B., 1983. A random walk-markow model for the distribution of time series. Journal of Business and Economics Statistics, 1(2): 169-173.

Wei, W.W.S. and D.O. Stram, 1990. Disaggregation of time series models. Journal of the Royal Statistical Society. Series B, 52(3): 453-467.