



Social Capital, Endogenous Labor Supply, and Economic Development

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Abstract

Social capital has been increasingly recognized as an important determinant of economic growth in the literature of economic growth. Nevertheless, there are only a few rigorous dynamic growth models which explicitly deal with dynamic interdependence between social capital, physical capital, and economic structure. The purpose of this study is to incorporate social capital in a neoclassical economic growth theory. We propose a dynamic model with interdependence between economic structural change, wealth accumulation, and social capital accumulation. Social capital positively affects total factor productivities and is accumulated through investment, leisure activities, and production. We simulate the model. The study focuses on effects of changes in some parameters on the equilibrium and transitional processes of the economic dynamics. We get some insights through including social capital in economic growth modelling. For instance, if society has lower trust (possibly if we interpret social capital as guanxi in Chinese societies) which results in a rise in depreciation rate of social capital, the economy suffers from falling social capital, productivities, national capital, and national output; consumers have lower income, wealth, and consumption; they also have to spend more time on investing in social capital.

Keywords: Social capital, Physical capital, Propensity to save, Propensity to accumulate social capital, Returns to scale in social capital, Economic structure, Chinese guanxi, Economic growth.

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Contribution of this paper to the literature

The paper makes a unique contribution to the literature of economic growth by introducing social capital accumulation to neoclassical growth theory. It shows a dynamic interdependence between social capital, physical capital, and economic structure.

1. Introduction

The purpose of this study includes endogenous social capital in neoclassical growth theory. Many economists consider social capital as an important determinant of economic growth (e.g., (Beugelsdijk & Van Schaik, 2005; Bofota, Boucekine, & Bala, 2016; Iyer, Kitson, & Toh, 2005; Knack & Keefer, 1997; Putnam, Leonardi, & Nanetti, 1993)). Many models have been developed to study relations between social capital and growth. The concept of social capital, according to Knack and Keefer (1997) refers to different meanings such as trust, cooperative, and associations within groups. Putnam et al. (1993) considers social capital as “those features of social organization, such as trust, norms, and networks, that can improve the efficiency of society by facilitating co-ordinated actions.” As far as how to model social capital is modelled, this study is influenced by Bofota et al. (2016). Different from Bofota et al. (2016) who include social capital in growth model by assuming that social capital affects human capital accumulation, this study assumes that social capital directly affects total factor productivities. Similar to the approach by Bofota et al. (2016) we assume that to accumulate social capital also involves a cost in terms of consumer goods.

The economic mechanism of growth is founded on the neoclassical growth theory (Burmeister & Dobell, 1970; Solow, 1956; Swan, 1956; Zhang, 2005). This paper is specially referred to Uzawa’s two-sector growth model (Uzawa, 1961). Different from the traditional Solow one-sector growth model, the Uzawa model deals with economic structural change by designing an economy with capital goods sector and consumer goods sector. Although this paper is developed within the Uzawa framework, we introduce an alternative approach to consumer decision by Zhang (2005) to examine behavior of households. We are concerned with dynamic interdependence between economic growth and social capital. As mentioned before, some recent endogenous growth models have emphasized the role of social capital as a determinant of economic growth. The rest paper is organized as follows. Section 2 defines the basic model of economic growth with inclusion of social capital. Section 3 shows how we can follow the movement of the economic system. Section 4 examines effects of changes in some parameters on the economic system over time. Section 5 concludes the study. The main results in Section 3 are shown in the Appendix.

2. The Growth Model with Social Capital

The basic features of the model are influenced by the growth model of Solow (1956) the two-sector growth model of Uzawa (1961) some growth models with endogenous social capital, and the approach to household decision by Zhang (1993). Similar to the Uzawa two-sector model, we consider an economy which is composed of capital goods and consumer goods sectors. We generalize the two-sector growth model by including endogenous social capital. Social capital affects productivities of production sectors. Social capital is accumulated through working, leisure activities, and individuals’ efforts. We choose the price of capital goods to be unity. Economic production is neoclassical. All markets are perfectly competitive. Capital and labor are freely mobile between sectors. We introduce the following variables. subscripts i and s – subscripts for the consumer goods sector and capital goods sector;

$N_j(t)$ and $K_j(t)$ – labor force and capital input employed by sector j at time t ;

$F_j(t)$ – the product level of sector j ;

$T(t)$, $\bar{T}(t)$, and $\hat{T}(t)$ – the representative household’s time spent on work, leisure, and accumulating social capital;

$p(t)$ – price of consumer good;

$w(t)$ and $r(t)$ – wage rate and rate of interest;

$\Lambda(t)$ – the representative household’s social capital;

$\bar{k}(t)$ – wealth owned by the representative household;

$c_i(t)$ and $c_s(t)$ – consumption of capital goods and consumption of consumer goods;

\bar{N} – fixed population;

δ_k and δ_Λ – fixed depreciation rates of physical capital and social capital, respectively.

2.1. Total Labor Supply

We use $N(t)$ to stand for the total labor supply, which is the total work hour by human capital as follows:

$$N(t) = h \bar{T}(t) \bar{N}, (1)$$

Where h is the fixed level of human capital.

2.2. Capital Goods Sector

We take the production function of the capital goods sector on the following form

$$F_i(t) = A_i(\Lambda(t)) K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \alpha_i, \beta_i > 0, \alpha_i + \beta_i = 1, (2)$$

where α_i , and β_i are parameters and $A_i(\Lambda(t))$ is the total factor productivity which is dependent on social capital. We do not specify how social capital affects the total factor productivity. Social capital is given for each firm. In perfectly competitive markets, labor and capital inputs are determined by their marginal products. The marginal conditions for the consumer goods sector are:

$$r_\delta(t) = \frac{\alpha_i F_i(t)}{K_i(t)}, w(t) = \frac{\beta_i F_i(t)}{N_i(t)}, (3)$$

where $r_\delta(t) \equiv r(t) + \delta_k$.

2.3. Consumer Goods Sector

The production function of the consumer goods sector is specified as:

$$F_s(t) = A_s(\Lambda(t)) K_s^{\alpha_s}(t) N_s^{\beta_s}(t), \alpha_s, \beta_s > 0, \alpha_s + \beta_s = 1, (4)$$

where α_s and β_s are parameters and $A_s(\Lambda(t))$ is the total factor productivity which is dependent on social capital. The marginal conditions imply:

$$r_\delta(t) = \frac{\alpha_s p(t) F_s(t)}{K_s(t)}, w(t) = \frac{\beta_s p(t) F_s(t)}{N_s(t)}. (5)$$

2.4. Disposable Income

This study the approach proposed by Zhang (1993; 2005) to model consumers' behavior. The current income is

$$y(t) = r(t) \bar{k}(t) + h w(t) T(t), (6)$$

where $r(t) k(t)$ is the interest payment and $h T(t) w(t)$ is the wage payment. The total value of the wealth that the household can sell to purchase goods and to save is $\bar{k}(t)$. The disposable income $\hat{y}(t)$ is the current income and the value of wealth. We have

$$\hat{y}(t) = y(t) + \bar{k}(t). (7)$$

2.5. Time Distribution and Budget

Denote the (fixed) available time for work, investment in social capital, and leisure by T_0 . The time distribution satisfies:

$$T(t) + \bar{T}(t) + \hat{T}(t) = T_0. (8)$$

The household spends the disposable income on consuming goods, accumulating social capital, and saving. We use $\omega(\Lambda(t))$ to measure cost per unit of time in terms of consumer goods in investing social capital. It is assumed that the cost is dependent on the stock of social capital. We don't specify function form of $\omega(\Lambda(t))$ at this stage of modelling. The household spends $\omega(\Lambda(t))\hat{T}(t)$ on accumulating human social capital and $s(t)$ on saving. The budget constraint is:

$$p(t) c_s(t) + c_i(t) + p(t) \omega(\Lambda(t)) \hat{T}(t) + s(t) = \hat{y}(t). (9)$$

Equation 9 implies that the consumption, investment, and saving use up the consumers' disposable income. Substitute (8) into (9):

$$p(t) c_s(t) + c_i(t) + h w(t) \bar{T}(t) + \bar{\omega}(t) \hat{T}(t) + s(t) = \bar{y}(t), (10)$$

where

$$\bar{y}(t) \equiv (1 + r(t))\bar{k}(t) + h T_0 w(t), \bar{\omega}(t) \equiv p(t) \omega(\Lambda(t)) + h w(t).$$

We interpret $\bar{y}(t)$ as the maximum disposable income for given wealth and human capital and $\bar{\omega}(t)$ as the opportunity cost for accumulating social capital.

2.6. Utility Function and Optimal Decision

We assume that utility level $U(t)$ of the representative household is dependent on $c_s(t)$, $c_i(t)$, $\bar{T}(t)$, $\hat{T}(t)$ and $s(t)$ as follows:

$$U(t) = c_s^{\gamma_0}(t) c_i^{\xi_0}(t) \bar{T}^{\sigma_0}(t) \hat{T}^{\theta_0}(t) s^{\lambda_0}(t), \sigma_0, \theta_0, \gamma_0, \xi_0, \lambda_0 > 0,$$

in which $\gamma_0, \xi_0, \sigma_0, \theta_0$, and λ_0 are the representative household's elasticity of utility with regard to consumer goods, capital goods, leisure time, time for accumulating social capital, and saving. We call $\gamma_0, \xi_0, \sigma_0, \theta_0$, and λ_0 propensities to consume good, to consume capital goods, to use leisure time, to use time for accumulating social capital, and to hold wealth, respectively. Maximizing $U(t)$ subject to the budget constraint yields:

$$c_s(t) = \frac{\gamma \bar{y}(t)}{p(t)}, c_i(t) = \xi \bar{y}(t), \bar{T}(t) = \frac{\sigma \bar{y}(t)}{h w(t)}, \hat{T}(t) = \frac{\theta \bar{y}(t)}{\bar{\omega}(t)}, s(t) = \lambda \bar{y}(t), (11)$$

where

$$\gamma \equiv \rho \gamma_0, \xi \equiv \rho \xi_0, \sigma \equiv \rho \sigma_0, \theta \equiv \rho \theta_0, \lambda \equiv \rho \lambda_0, \rho \equiv \frac{1}{\gamma_0 + \xi_0 + \sigma_0 + \theta_0 + \lambda_0}.$$

2.7. Wealth Accumulation

The change in wealth is the saving minus the dissaving. According to the definitions of $s(t)$ and $\bar{k}(t)$, wealth changes according to the following accounting equation:

$$\dot{\bar{k}}(t) = s(t) - \bar{k}(t). (12)$$

2.8. Full Employment of Capital and Labor

The total capital stocks $K(t)$ is fully employed. We have:

$$K_i(t) + K_s(t) = K(t), N_i(t) + N_s(t) = N(t). (13)$$

2.9. Demand of and Supply for Two Goods

The equilibrium condition for consumer goods implies:

$$c_s(t) \bar{N} + \omega(\Lambda(t)) \hat{T}(t) \bar{N} = F_s(t). (14)$$

We have the equilibrium condition for capital goods as follows:

$$c_i(t) \bar{N} + s(t) \bar{N} + \delta_k K(t) = F_i(t) + \bar{k}(t) \bar{N}. (15)$$

2.10. Wealth Owned by the Households and National Output

The national wealth is held by the households:

$$\bar{k}(t) \bar{N} = K(t). (16)$$

The national output $Y(t)$ and output per household $f(t)$ are:

$$Y(t) = F_i(t) + p(t) F_s(t), f(t) \equiv \frac{Y(t)}{N}. \quad (17)$$

2.11. Dynamics of Social Capital

This study treats social capital as capital such as physical capital and human capital. Solow (1995) thinks that it is difficult to treat social capital because the measurement of its stock seems far away. After reviewing the literature on social capital, Chou (2006) argues that social capital has some similarities to physical and human capital with regard to intertemporal dimension and ability to create a stream of future benefits. This study treat social capital as capital Social capital is affected by different channels. Social capital is accumulated through investment, learning by producing, and leisure activities. This study proposes the following social capital accumulation equation

$$\dot{\Lambda}(t) = \frac{u_s \hat{T}^{a_s}(t)}{\Lambda^{\chi_s}(t)} + \frac{u_i f^{a_i}(t)}{\Lambda^{\chi_i}(t)} + \frac{u_c \bar{T}^{a_c}(t)}{\Lambda^{\chi_c}(t)} - \delta_{\Lambda} \Lambda(t), \quad (18)$$

where u_m , χ_m , and a_m , $m = s, i, c$, are parameters. We require u_m and u_a non-negative. Signs of χ_m are not specified. The term $u_s \hat{T}^{a_s}/\Lambda^{\chi_s}$ implies that a rise in hours spent on social capital augments social capital and the accumulation exhibits decreasing (increasing) of return to scales if $\chi_s > (<)0$. The term $u_i f^{a_i}/\Lambda^{\chi_i}$ implies accumulating social capital through producing. A higher per capita output enhances increase social capital accumulation. The term $u_c \bar{T}^{a_c}/\Lambda^{\chi_c}$ implies a positive relationship between leisure time and social capital accumulation with decreasing (or increasing) return to scale.

We built the dynamic growth model with endogenous social capital.

3. The Dynamics of the Economy

The appendix shows that the movement of the national economy can be described by two differential equations with $z(t)$ and $\Lambda(t)$ as the variables, where $z(t) \equiv w(t)/(r(t) + \delta_k)$. The following lemma shows how we follow the movement of all the variables of the dynamic system.

3.1. Lemma

We can determine the movement of $\bar{k}(t)$ and $\Lambda(t)$ is by the following two differential equations

$$\dot{\Lambda}(t) = \Omega_{\Lambda}(z(t), \Lambda(t)), \dot{z}(t) = \Omega_z(z(t), \Lambda(t)), \quad (19)$$

where Ω_{Λ} and Ω_z are functions of $\Lambda(t)$ and $z(t)$ defined in the Appendix. We have all the other variables as functions of $\Lambda(t)$ and $z(t)$ as follows: $r(t)$ and $w(t)$ by (A2) $\rightarrow p(t)$ by (A3) $\rightarrow \bar{k}(t)$ by (A13) $\rightarrow \bar{y}(t)$ by (A4) $\rightarrow K(t)$ by (16) $\rightarrow N_s(t)$ by (A6) $\rightarrow T(t)$ by (A7) $\rightarrow N(t)$ by (A8) $\rightarrow N_i(t)$ by (A9) $\rightarrow K_i(t)$ and $K_s(t)$ by (A1) $\rightarrow F_i(t)$ by (2) $\rightarrow F_s(t)$ by (4) $\rightarrow c_{j_s}(t)$, $c_i(t)$, $\bar{T}(t)$ and $\hat{T}(t)$ by (11).

The lemma shows how to determine movement of all the variables once we determine the movement of the two state variables, $\Lambda(t)$ and $z(t)$. We simulate the model by specifying parameter values as follows:

$$\begin{aligned} N = 100, T_0 = 24, h = 2.5, \alpha_i = 0.3, \alpha_s = 0.33, \lambda_0 = 0.7, \xi_0 = 0.08, \gamma_0 = 0.06, \\ \theta_0 = 0.02, \sigma_0 = 0.2, \chi_s = 0.5, \chi_i = 0.6, \chi_c = 0.5, u_s = 0.4, u_i = 0.3, u_c = 0.2, \\ a_s = 0.3, a_i = 0.2, a_c = 0.1, \delta_k = 0.05, \delta_{\Lambda} = 0.05. \end{aligned} \quad (20)$$

The population is 100. The propensity to save is 0.7. The propensity to consume consumer goods and to consume capital goods are respectively 0.06 and 0.08. The depreciation rates of physical capital and social capital are fixed 5 percent. The social capital accumulation exhibits decreasing returns to scale in leisure, production and investment. Human capital is fixed at 2.5. We specify the total factor productivity functions of the two sectors as follows:

$$A_i(t) = 0.7e^{0.2\Lambda(t)-0.1\Lambda^{1.2}(t)}, A_s(t) = 0.7e^{0.2\Lambda(t)-0.1\Lambda^{1.2}(t)}.$$

The relationships between social capital and total factor productivities are plotted in Figure 1. For low levels of social capital productivities rise in association with increases in social capital; for high levels of social capital productivities fall as social capital rises. If we interpret social capital as, for instance, Chinese *guanxi*, high complexity of *guanxi* might not enhance productivities. We specify only a special case about how social capital may affect productivities.

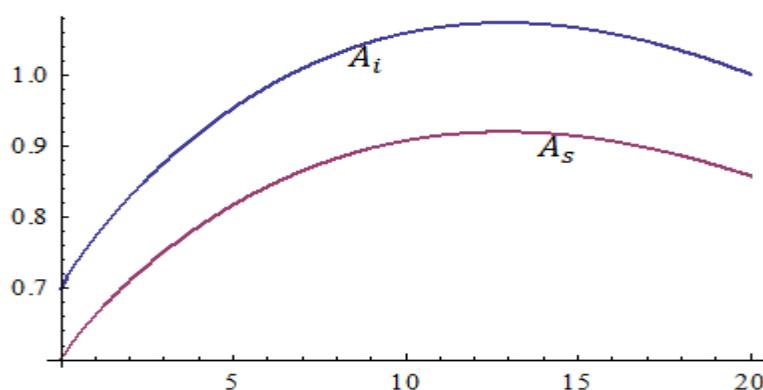


Figure-1. Relationship between social capital and total factor productivities.

We specify the cost function as follows:

$$\omega(\Lambda(t)) = 0.4\Lambda^{1.2}(t).$$

It implies that cost rises as social capital becomes higher. We choose the initial conditions

$$z(0) = 0.08, \Lambda(0) = 8.$$

We plot the movement of the national economy in Figure 2. Social capital and physical capital fall. The productivities of the two sectors fall. The wage rate falls, and rate of interest rises. The national labor force rises. The labor input of the consumer goods sector falls and the labor input of the capital goods sector rises. The household spends less hours on social capital. The household's consumption levels of two goods and wealth fall.

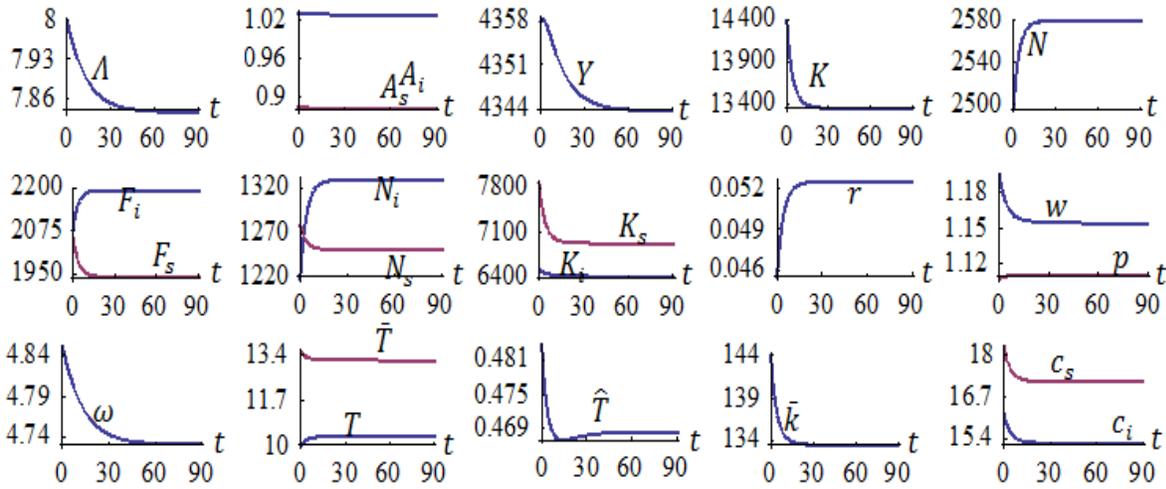


Figure-2. The motion of the national economy.

We calculate the equilibrium values of the variables as follows

$$\Lambda = 7.84, Y = 4344, N = 2579, K = 13341, F_i = 2192, F_s = 1938, K_i = 6414, K_s = 6927, N_i = 1330, N_s = 1250, A_i = 1.03, A_s = 0.88, r = 0.053, w = 1.15, p = 1.11, \omega = 4.73, \bar{k} = 133, c_i = 15.3, c_s = 17.2, T = 10.3, \bar{T} = 13.2, \hat{T} = 0.47.$$

We calculate eigenvalues to examine stability. The two eigenvalues are respectively -0.257 and -0.08 . The equilibrium point is stable.

4. Comparative Dynamic Analysis

The movement of the variables was plotted in the previous section. It is straightforward to follow the movement of the system as any parameter is changed. This section conducts comparative dynamic analysis with regards different exogenous changes. We introduce a variable, $\bar{\Delta}x(t)$, to represent for the change rate of the variable $x(t)$ in percentage due to changes in the parameter value.

4.1. Negative Effects of Social Capital Being Strengthened

First, we study what happen to the economic system when the total factor productivity functions are changed as follows:

$$A_i(t) = 0.7e^{0.2\Lambda(t)-0.1\Lambda^{1.2}(t)} \Rightarrow 0.7e^{0.2\Lambda(t)-0.105\Lambda^{1.2}(t)},$$

$$A_s(t) = 0.6e^{0.2\Lambda(t)-0.1\Lambda^{1.2}(t)} \Rightarrow 0.6e^{0.2\Lambda(t)-0.105\Lambda^{1.2}(t)}.$$

The negative effects of social capital on the total factor productivities are strengthened. The changes in the movement of the economic system are plotted in Figure 3. The stock of social capital and the total factor productivities of the two sectors factor fall. The national output and national capital are all reduced. The total labor supply falls initially and rises slightly in the long term. The household spends less time on social capital accumulation. The price of services is slightly affected. The wage rate is reduced. The per unit of time cost of social accumulation falls. The consumption level and wealth are reduced. As the negative impact of social capital on productivities become more dominant, the economic growth will suffer.

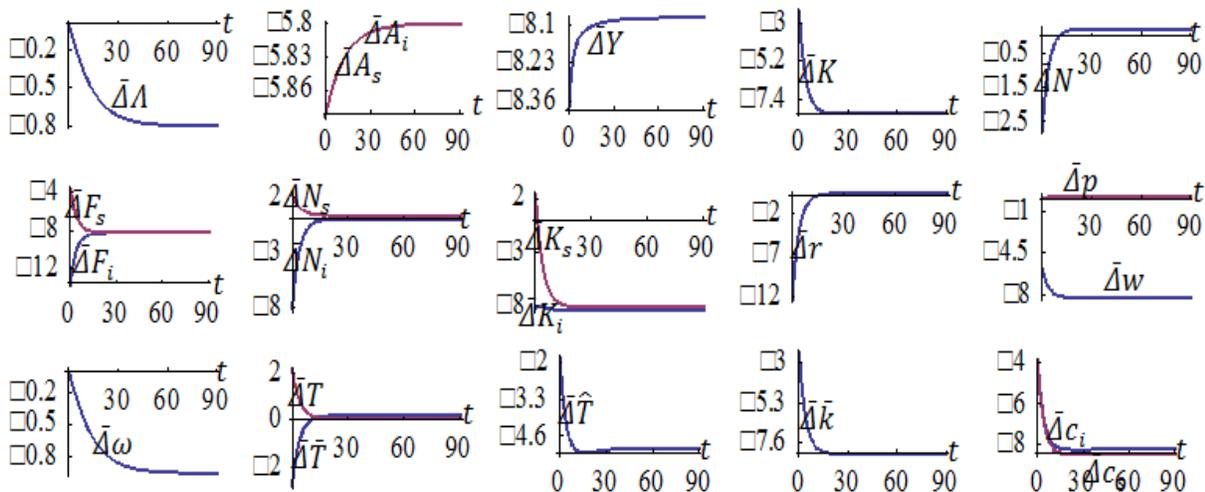


Figure-3. Negative effects of social capital being strengthened.

4.2. Human Capital Being Increased

We now study what happen to the economic system when human capital is increased as follows: $h: 2.5 \Rightarrow 2.6$. The result is plotted in Figure 4. The stock of social capital is increased. The household spends more hours on accumulating social capital and less hours on leisure and working. The national wealth, national output and total labor supply are enhanced. The total factor productivities are augmented. The two sectors are expanded. All the inputs are increased. The rate of interest falls. The price of consumer goods falls slightly. The wage rate is increased. The per unit time cost of accumulating social cost is increased. The per household wealth and consumption levels are enhanced.

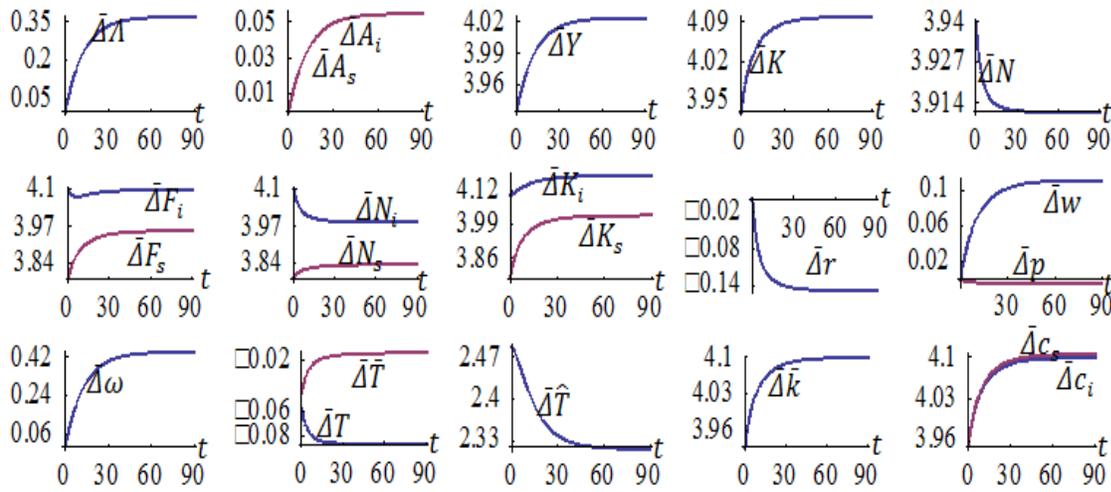


Figure-4. Human capital being increased.

4.3. The Propensity to Accumulate Social Capital Rises

We now allow the propensity to accumulate social capital to be enhanced as follows: $\theta_0: 0.02 \Rightarrow 0.022$. We plot the simulation result in Figure 5. The stock of social capital is increased. The household spends more hours on accumulating social capital and less hours on leisure. The household works less hours initially and less in the long term. The national wealth and national output fall. The total labor supply falls initially and rises in the long term. The total factor productivities are augmented. The consumer goods sector shrinks and the capital goods sector expands. The service sector's capital input changes slightly and labor input rise. The two inputs of the capital goods sector are augmented. The wage rate falls. The price of consumer goods and rate of interest rise. The per unit time cost of accumulating social cost is enhanced. The per household wealth and consumption levels of two goods are enhanced.

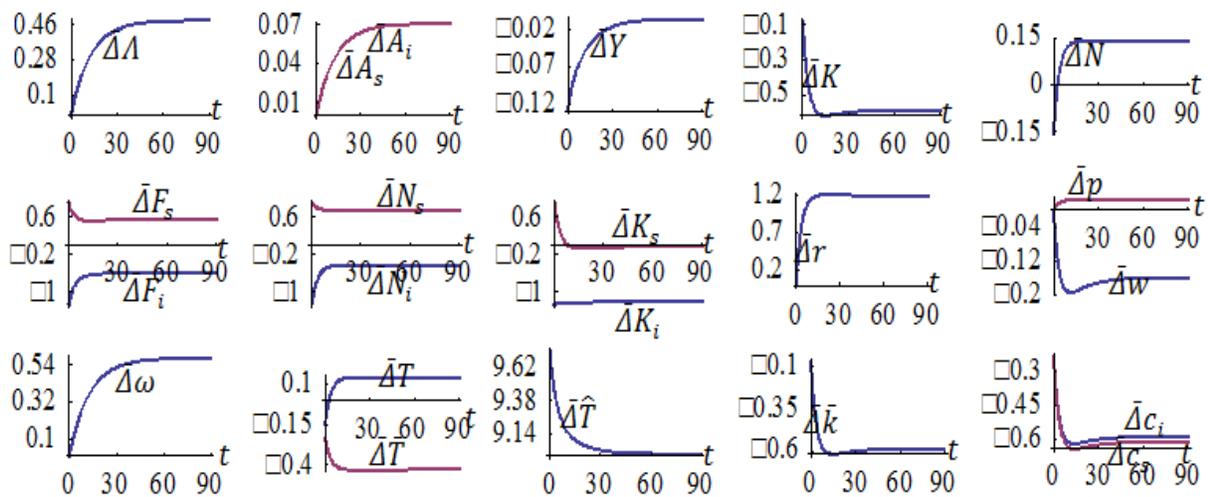


Figure-5. The propensity to accumulate social capital rises.

4.4. The Cost of Social Accumulation Rises

We study the movement of economic system when the cost function of social capital accumulation changes as follows:

$$\omega(\lambda(t)) = 0.4\lambda^{1.2}(t) \Rightarrow 0.45\lambda^{1.2}(t).$$

We plot the simulation result in Figure 6. For a given level of social capital, it costs more for the household to accumulate social capital. Opportunity cost of accumulating human capital rises. The stock of social capital and total factor productivities fall. The household spends less hours on accumulating social capital and more hours on leisure and work. The national output rises. The national capital rises initially and falls in the long term. The wage rate falls. The price of consumer goods and rate of interest rise. The total labor supply rises. The capital goods sector shrinks initially and expands in the long term. The consumer goods sector expands. The consumer (capital) goods sector's capital input rise (falls). The per household wealth and consumption levels of two goods rise initially and fall in the long term.

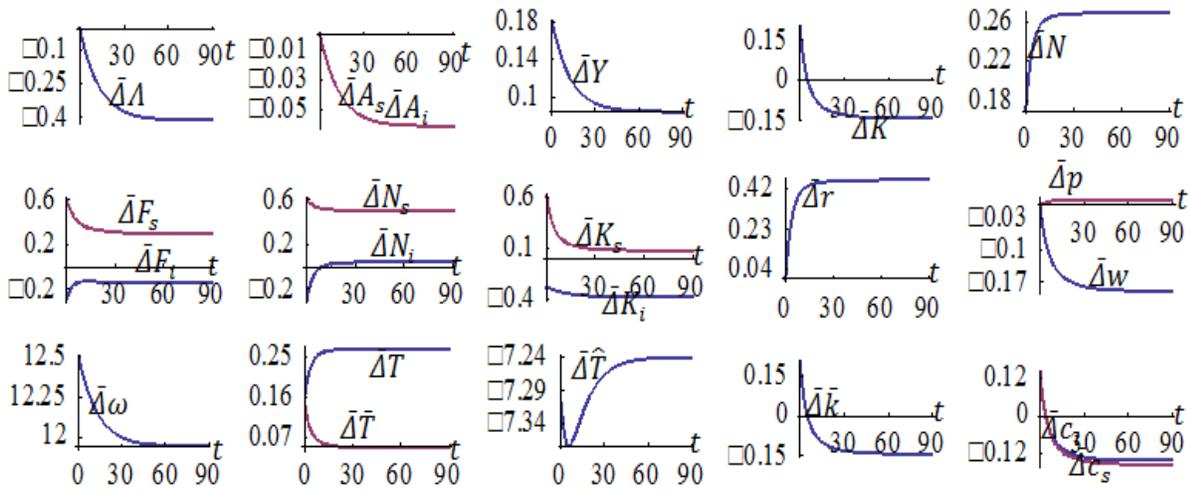


Figure-6. The cost of social accumulation rises.

4.5. The Depreciation Rate of Social Capital Rises

We deal with the impact of the following fall in the depreciation rate of social capital $\delta_A: 0.05 \Rightarrow 0.06$. We plot the simulation result in Figure 7. The stock of social capital and total factor productivities fall. The household spends more hours on accumulating social capital and less hours on work. Leisure time is slightly affected. The national output, national capital and national output fall. The wage rate and rate of interest fall. The price of consumer goods slightly changes. The capital goods sector shrinks initially and expands in the long term. The two sectors shrink. The per household wealth and consumption levels of two goods fall.

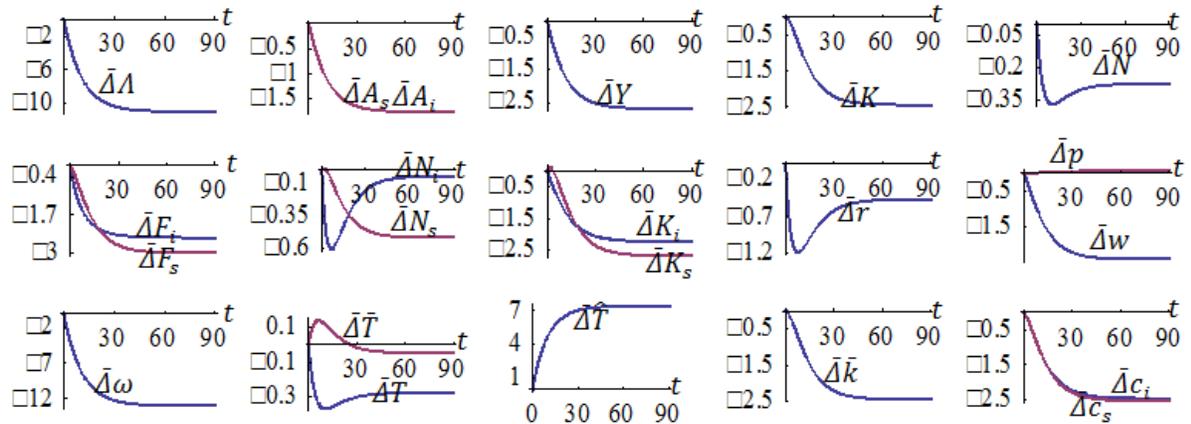


Figure-7. The depreciation rate of social capital rises.

4.6. The Propensity to Accumulate Wealth Rises

We consider the case that the propensity to accumulate wealth rises as follows: $\lambda_0: 0.7 \Rightarrow 0.71$. We plot the simulation result in Figure 8. The household has more wealth and the economy has more capital. The stock of social capital and total factor productivities are enhanced. The household spends less hours on accumulating social capital in the short term and more in the long term. The household works more hours and has less leisure time. The national output and national labor input rise. The consumer goods sector shrinks initially and expands in the long term. The capital goods sector expands. The wage rate is enhanced. The price of consumer goods and rate of interest fall. The per unit time cost of accumulating social cost is enhanced. The consumption levels of two goods fall initially and rise in the long term.

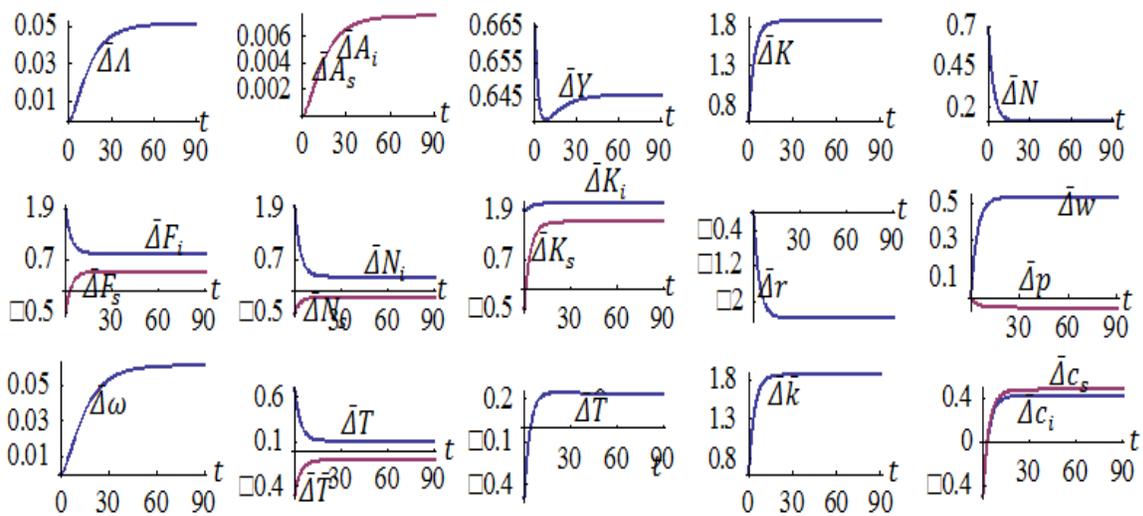


Figure-8. The propensity to accumulate wealth rises.

4.7. Social Capital Has Lower Return to Scale in Accumulating through Producing

We study the case that social capital has lower return to scale in accumulating through producing as follows: $\chi_i: 0.6 \Rightarrow 0.7$. We plot the simulation result in Figure 9. The stock of social capital and total factor productivities fall. The household spends more hours on accumulating social capital and less hours on work. Leisure time is slightly affected. The national output, national capital and national output fall. The wage rate and rate of interest fall. The price of consumer goods slightly changes. The two sectors shrink. The per household wealth and consumption levels of two goods fall.

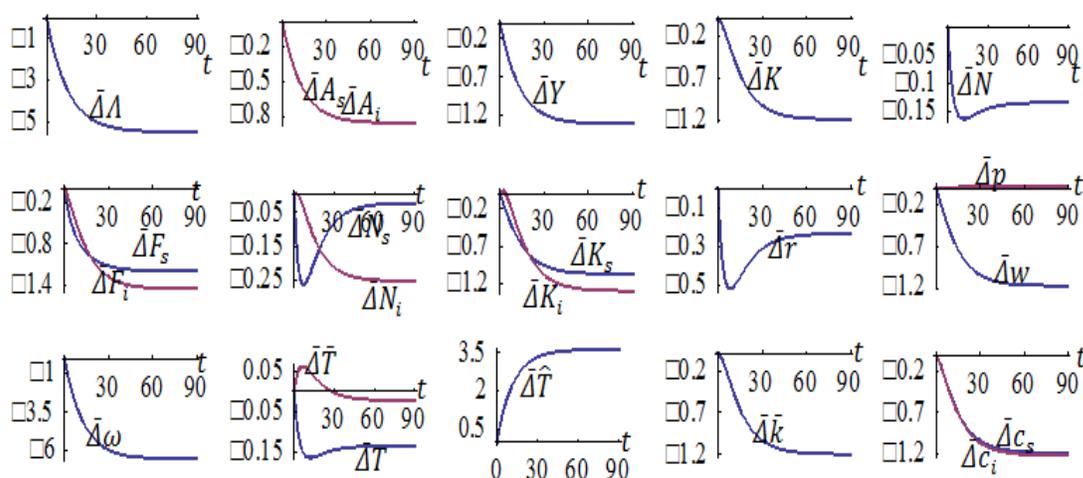


Figure-9. Social capital has lower return to scale in accumulating through producing.

5. Conclusions

This paper constructed a neoclassical two-sector economic growth model with endogenous social capital in a competitive economy. The model describes nonlinear dynamic interdependence between economic structural change, wealth accumulation, and social capital accumulation under perfectly competitive conditions. This study assumes that social capital positively affects total factor productivities. We simulate the model. The model has a stable unique equilibrium point for the specified parameter values. The study focuses on effects of changes in some parameters on the equilibrium and transitional processes of the economic dynamics. We get some insights into interdependence between social capital and economic growth. For instance, if society has lower trust (possibly like in Chinese societies when we interpret social capital as *guanxi*) which results in a rise in depreciation rate of social capital, the economy suffers from falling social capital, productivities, national capital, and national output; consumers have lower income, wealth, and consumption; they also have to spend more time on investing in social capital. The study can be extended and generalized in different ways. It is straightforward to do more comparative dynamic analysis. There are many models with extensions and generalizations of the Solow model and the Uzawa two-sector growth model. It is possible to further develop our model on basis of the literature.

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Appendix: Proving the Lemma

We now confirm the lemma in section 3. From (3) and (5), we obtain

$$z \equiv \frac{r_\delta}{w} = \frac{\tilde{\alpha}_i N_i}{K_i} = \frac{\tilde{\alpha}_s N_s}{K_s}, \quad (A1)$$

where we omit time index and $\tilde{\alpha}_i \equiv \tilde{\alpha}_j / \tilde{\beta}_j, j = i, s$. Insert (2) and (A1) in (3)

$$r_\delta = \frac{\alpha_i A_i z^{\beta_i}}{\tilde{\alpha}_i^{\beta_i}}, w = \frac{\beta_i A_i \tilde{\alpha}_i^{\alpha_i}}{z^{\alpha_i}}. \quad (A2)$$

We express $\bar{\Delta}K_s$ and $\bar{\Delta}K_i$ as functions of Λ and $\bar{\Delta}k_2$ Equations (A1), (4) and (5) imply

$$p = \frac{\tilde{\alpha}_s^{\beta_s} r_\delta}{\alpha_s A_s z^{\beta_s}}. \quad (A3)$$

We also have

$$\bar{y} = R \bar{k} + h T_0 w, \quad (A4)$$

where $R \equiv 1 + r$. Insert (11) in (14)

$$\left(\gamma \bar{N} + \frac{\theta \bar{N} p \omega}{\bar{\omega}} \right) \bar{y} = p F_s. \quad (A5)$$

Insert (5) in (A5)

$$N_s = R_s \bar{y}, \quad (A6)$$

where

$$R_s(z, \Lambda) \equiv \left(\gamma \bar{N} + \frac{\theta \bar{N} p \omega}{\bar{\omega}} \right) \frac{\beta_s}{w}.$$

From (8) and (11) we have:

$$T = T_0 - \frac{\sigma \bar{y}}{hw} - \frac{\theta \bar{y}}{\bar{\omega}}.$$

Insert (A6) in (1)

$$N = T_0 h \bar{N} - R_N \bar{y}, \quad (A8)$$

where

$$R_N \equiv \left(\frac{\sigma \bar{y}}{hw} + \frac{\theta}{\bar{\omega}} \right) h \bar{N}.$$

Equations (13), (A8) and (A6) imply

$$N_i = T_0 h \bar{N} - (R_N + R_s) \bar{y}, \quad (A9)$$

Equations (13) and (16) imply:

$$\bar{k} \bar{N} = K_i + K_s. \quad (A10)$$

Insert (A1) in (A10)

$$z \bar{k} \bar{N} = \tilde{\alpha}_i N_i + \tilde{\alpha}_s N_s. \quad (A11)$$

Insert (A6) and (A9) in (A11)

$$z \bar{k} = \tilde{\alpha}_i h T_0 - \bar{R} \bar{y}, \quad (A12)$$

where

$$\bar{R} \equiv \frac{\tilde{\alpha}_i R_N + \tilde{\alpha}_i R_s - \tilde{\alpha}_s R_s}{\bar{N}}.$$

Insert (A4) in (A12)

$$\bar{k} = \Psi(z, \Lambda) \equiv \frac{\tilde{\alpha}_i h T_0 - h T_0 w \bar{R}}{z + \bar{R} R}. \quad (A13)$$

It is straightforward to check that all the variables can be expressed as functions of z and Λ at any point in time as follows: r and w by (A2) $\rightarrow p$ by (A3) $\rightarrow \bar{k}$ by (A13) $\rightarrow \bar{y}$ by (A4) $\rightarrow K$ by (16) $\rightarrow N_s$ by (A6) $\rightarrow T$ by (A7) $\rightarrow N$ by (A8) $\rightarrow N_i$ by (A9) $\rightarrow K_i$ and K_s by (A1) $\rightarrow F_i$ by (2) $\rightarrow F_s$ by (4) $\rightarrow c_{js}, c_i, \bar{T}$ and \hat{T} by (11). From this procedure, (12) and (17) we have:

$$\dot{\bar{k}} = \Omega_0(z, \Lambda) \equiv \lambda \bar{y} - \bar{k}, \quad (A14)$$

$$\dot{\Lambda} = \Omega_\Lambda(z, \Lambda) \equiv \frac{u_s \hat{T}^{a_s}}{\Lambda \chi_s} + \frac{u_i f^{a_i}}{\Lambda \chi_i} + \frac{u_c \bar{T}^{a_c}}{\Lambda \chi_c} - \delta_\Lambda \Lambda. \quad (A15)$$

Take derivatives of (A13) with respect to time

$$\dot{\bar{k}} = \frac{\partial \bar{k}}{\partial z} \dot{z} + \frac{\partial \bar{k}}{\partial \Lambda} \dot{\Lambda}. \quad (A16)$$

Insert (A14) and (A15) in (A16)

$$\dot{z} = \Omega_z(z, \Lambda) \equiv \left(\Omega_0 - \frac{\partial \bar{k}}{\partial \Lambda} \Omega_\Lambda \right) \left(\frac{\partial \bar{k}}{\partial z} \right)^{-1}. \quad (A17)$$

We thus proved the Lemma.