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Collaborative inquiry-based instructional model to enhance mathematical analytical thinking and reasoning skills for fourth-grade students

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Abstract

In contemporary society, students' analytical thinking and reasoning skills are crucial for their future development. The fourth grade (ages 9-10) is a critical stage for cultivating higher-level cognitive skills in students. Mathematics plays a significant role during this period, serving as a fundamental tool for fostering students' analytical thinking and reasoning abilities. Collaborative inquiry-based learning is considered beneficial for enhancing students' analytical thinking and reasoning skills. Therefore, this study aims to design a collaborative inquiry-based instructional model based on the characteristics of mathematics textbooks. The study extensively analyzes primary school mathematics textbooks published by Beijing Normal University and integrates the essential elements of collaborative inquiry-based learning to devise an instructional model tailored to the characteristics of mathematics textbooks. The model comprises six parts: Contextual Introduction (Question 1), Collaborative Inquiry (Question 2), Communication and Sharing (First round), Collaborative Inquiry (Question 3), Communication and Sharing (Second round), Review and Conclusion. The model is designed to assist students in acquiring knowledge through collaborative inquiry, stimulate their curiosity through contextualized learning, improve their analytical thinking and reasoning skills through collaborative inquiry, foster cognitive development through effective communication and sharing, and ultimately solidify acquired knowledge through reflection and conclusion. Theoretically, the analysis suggests that this model may positively impact students' analytical thinking and reasoning abilities, providing a valuable reference for practical teaching.

Keywords: Analytical thinking, Collaborative inquiry-based learning, Instructional model, Mathematical curriculum, Primary education, Reasoning skills.

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Contents

Contribution of this paper to the literature
This study contributes to the existing literature by presenting a tailored collaborative inquiry-based learning instructional model for fourth-grade mathematics instruction. The proposed model emphasizes the importance of contextualized learning, inquiry-based learning, effective communication, and reflective conclusion in fostering cognitive development during this critical stage of education.

1. Introduction

Reasoning and analytical thinking skills are widely recognized as one of the essential competencies of the 21st century (Binkley et al., 2012; National Research Council, 2012). These abilities not only help individuals solve problems better, but also develop critical thinking and innovative skills that make them competitive in a changing society and workplace (Ahmad et al., 2017; Cobo, 2013). Therefore, enhancing the reasoning and analytical thinking skills of students has become an essential task in education.

Collaborative inquiry-based learning is an effective instructional method that promotes the development of students' reasoning and analytical thinking skills (Csanadi, Kollar, & Fischer, 2021; Ramadani, Supardi, & Hariyono, 2021; Sen, Ay, & Güler, 2021). Through collaborative inquiry, students are able to communicate and work with each other in tiny groups to solve problems together (Bell, Urhahne, Schanze, & Ploetzner, 2010; Nelson, 1999). This mode of instructional stimulates students' thinking and creativity, and develops their collaborative and problem-solving skills (Agbi & Yuangsoi, 2022).

Implementing learning in the 4th grade math course is significant. First, 4th grade is a critical period of rapid cognitive development for students who already have a mathematical foundation that allows them to reason and analyze in greater depth (Gale, O'Callaghan, Godfrey, Law, & Martyn, 2004). Second, the math course places greater demands on students' reasoning and analytical thinking skills (Hasanah, Tafrilyanto, & Aini, 2019). Since math is a subject that requires logical thinking and reasoning skills (Lusyana & Wangge, 2016) students' reasoning and analytical skills can be better developed through the implementation of collaborative inquiry-based learning in the math course (Gillies, Nichols, Burgh, & Haynes, 2014; Yumiati & Noviyanti, 2017).

However, there is still a gap in research on how to implement collaborative inquiry-based learning in the 4th grade math courses. While there have been some studies focusing on the effects of collaborative inquiry-based learning on the reasoning and analytical thinking skills of students, these studies have not involved models of collaborative inquiry-based learning that can be replicated or learned from. The objective of this study, therefore, is to fill this gap by formulating a collaborative inquiry-based instructional model derived from the principles of collaborative inquiry-based learning and the characteristics of the mathematics textbook. This endeavour aims to enhance the logical and analytical cognitive abilities of fourth-grade pupils and to contribute to the existing body of knowledge on collaborative inquiry-based education.

2. Literature Review

2.1. Collaborative Inquiry-based Learning (CIBL)

CIBL allows multiple students to actively collaborate on their inquiry-based questions. CIBL provides an opportunity for students to work collaboratively to complete and answer inquiry questions in ways that are not possible for individual students. With the support of the group members, students are able to complete learning tasks in their zone of proximal development (Fernández, Wegerif, Mercer, & Rojas-Drummond, 2015).

Teachers should begin by explaining the task requirements to students and underscore the importance of collaboration. Throughout the collaborative inquiry-based learning process within small groups, the teacher offers support to the group only when necessary (Urhahne, Schanze, Bell, Mansfield, & Holmes, 2010). As per the request, the CIBL group initiated the completion of the assignment. Students within each CIBL group should leverage their diverse talents to address most challenges that may arise in the learning process. In CIBL, the teacher should refrain from actively scaffolding the CIBL group. Instead, they should monitor the progress of each CIBL group and provide guidance where necessary (Chin & Hortin, 1993; Cifuentes, 1997), motivate an inactive CIBL group, and offer feedback after completing specific CIBL stages (Hsiao, Hong, Chen, Lu, & Chen, 2017; Williams, Nguyen, & Mangan, 2017; Zheng & Zhuang, 2008).

CIBL empowers students to acquire knowledge in domains relevant to the inquiries being posed and provides numerous opportunities for students to cultivate and exercise skills essential for the 21st century. The Alliance for 21st Century Skills is a framework developed in collaboration between the government and businesses to foster the advancement of proficiencies, aptitudes, and dispositions crucial for success in the workforce and modern society (Framework for 21st Century Learning, 2019). Three classifications of capabilities have been identified: (1) erudition skills, encompassing originality, novelty, evaluative reasoning, problem-solving, communication, and cooperation; (2) literacy skills, including informational literacy, media literacy, and ICT literacy (Information and Communication Technology (ICT) literacy refers to the ability to use, manage, and evaluate information and communication technologies effectively); and (3) vital life skills, which entail adaptability, versatility, initiative, self-guidance, communal and intercultural competencies, productivity, accountability, as well as leadership and responsibility.

Research has demonstrated that the implementation of CIBL not only yields a favorable influence on students' academic accomplishments (Kolloffel, Eysink, & De Jong, 2011; Lämsä, Hämäläinen, Koskinen, & Viiri, 2018; Pifarré & Staarman, 2011) but also reinforces high-level cognitive skills, including analytical thinking and reasoning abilities, to better adapt to society (Lu, Pang, & Shadiev, 2021).

2.2. Analytical Thinking

Analytical thinking skills, as an important component of higher-order thinking skills (Dillon & Scott, 2002; Miri, David, & Uri, 2007; Zohar & Dori, 2003) help improve student academic performance, reduce weaknesses, interpret, synthesize, solve problems, and control information, ideas, and daily activities (Ahmad et al., 2017). Analytical thinking is a extremely effective cognitive tool for understanding the components of a situation. It can be described as the skill of examining and deconstructing facts and ideas, assessing their merits and flaws. Cultivating this ability helps in problem solving, analyzing data, retrieving and applying information (Amer, 2005). Based on this interpretation, analytical thinking is characterized by the advantage of a systematic approach (Fabio & Towey, 2018). Moreover, analytical thinking implies logical connections by encoding reality into abstract symbols, words, or numbers (Iannello & Antonietti, 2008).

Analytical thinking is characterized by pre-analytical, analytical-partial, semi-analytical and complete analysis (Parta, 2016). Parta considers pre-analytical thinking is characterized by the use of standard procedures that are inappropriate for the problem conditions, leading to illogical steps such as dividing by zero. Another indication of pre-analytical thinking is the use of surface information, such as sketching a graph. Partial analytic thinking is characterized by the discontinuance of one part of the analytical procedure and the use of alternative methods. Semi-analytical thinking is characterized by "turning" and "vague" procedures or duplication of procedures. Analytical thinking is characterized by clear algorithms, logical reasoning, and essential statements underlying the process. The study also found that 33 percent of the subjects still exhibited pre-analytical thinking in solving simple problems, indicating the need for innovative teaching approaches to ensure better internalization of knowledge and learning experiences (Parta, 2016). Thus, collaborative inquiry-based learning can be an effective instructional method for improving analytical thinking.

2.3. Reasoning Skills

Deduction, inference, and logical cogitation formed upon accessible data, erudition, and logical associations represent the aptitude of reasoning (Kyllonen, 2020). It constitutes a cognitive faculty of elevated caliber that encompasses the procedure of deducing novel corollaries or resolving predicaments through logical ratiocination and ratiocination derived from established facts or premises. Reasoning proficiencies facilitate individuals in unearthing correlations and configurations amidst entities, thus enabling them to formulate judicious assessments and determinations.

Reasoning skills encompass the cognitive processes essential for problem-solving in various domains (Henderson et al., 2001). Keazer and Menon (2015) emphasized reasoning skills are widely acknowledged as the foundation of mathematical proficiency, and their absence can disrupt mathematics education. Since reasoning underpins all mathematical principles and operations, reasoning skills are considered the cornerstone of the field of mathematics (Aysun & Yildiz, 2005; Umay, 2003).

2.4. Current Status of the Study

Collaborative inquiry-based learning has been confirmed by current research to have a positive effect on enhancing students' analytical thinking and reasoning skills (Ramadani et al., 2021; Sen et al., 2021). While inquiry-based learning modalities have been well studied (Al Mamun, Lawrie, & Wright, 2020; Bybee, Taylor, Gardner, Westbrook, & Landes, 2006; Pedaste et al., 2015) there has been relatively limited work on collaborative inquiry-based learning modalities. As a scientifically valid mode of teaching, collaborative inquiry-based learning is applicable to teaching across disciplines. However, when combining it with a specific discipline, it should not be applied directly, collaborative inquiry-based learning should be integrated with the content of the discipline to better translate the theory into practice. Therefore, the aim of this study is to develop a collaborative inquiry-based instructional model that integrates collaborative inquiry-based learning with mathematics textbook for Grade 4.

3. Method

3.1. Teaching Materials

The mathematics textbook used in this study is "Fourth Grade Mathematics, Volume One", which is approved by the Ministry of Education of China and published by Beijing Normal University (BUN).

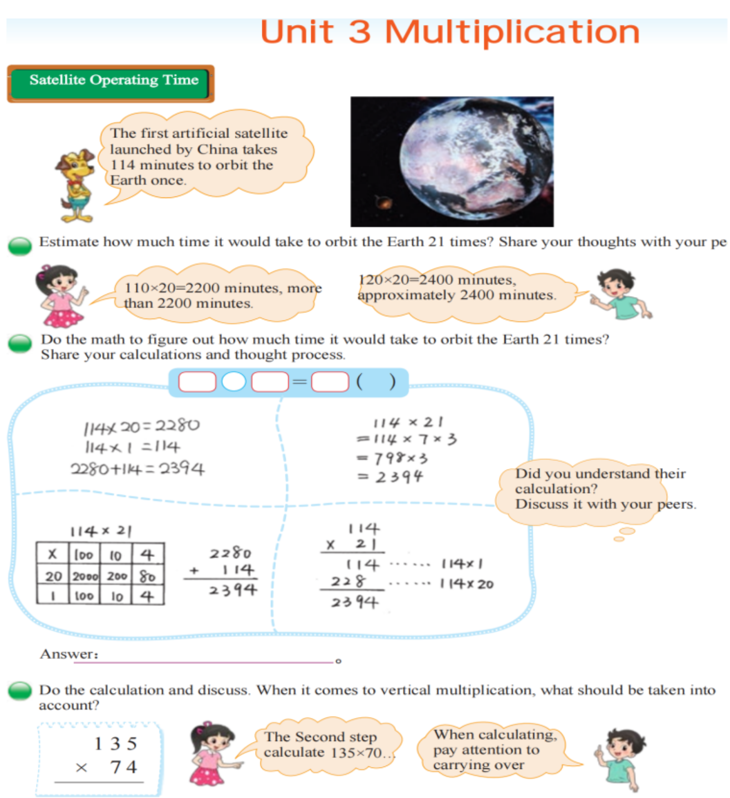

Figure 1. Fourth-grade mathematics textbook lesson sample.

The mathematics textbook shows the nature of mathematics as originating from and applied to life, with its unique features of life, activities and problems. When you open the BNU primary mathematics textbook, the first thing that hits you are the vivid, interesting, and colorful illustrations that are closely related to the lives of the students. These scenarios are practical items that students can access in their daily lives and are close to the student's life, such as bus problems, shopping problems, etc., so that mathematics is closely related to reality.

In terms of content arrangement, the BNU primary mathematics textbook uses a unique format that avoids the use of examples and instead allows students to experience and learn mathematics through engaging activities such as recognizing, talking, doing, counting, comparing and arranging. The learning of each knowledge point is guided by mathematical activities, and the teaching process essentially follows the basic narrative pattern of "problem situation - modelling - interpretation and application".

Further, the BNU primary mathematics textbook is problem-oriented, with green dots as questions, and these questions are interleaved to form a string of questions that together build a vertical extension of knowledge depth. This design not only reflects the systematic nature of the textbook, but also prompts students to develop their thinking and analytical skills during the problem solving process.

3.2. Research Framework

Collaborative inquiry-based learning is an approach that emphasizes collaboration and interaction among students, promoting the building of knowledge and deepening of understanding through joint thinking, discussion, and problem-solving. In collaborative inquiry learning, students take an active role, collaborating with others in exploring and discovering knowledge, rather than just passively accepting a teacher's knowledge transfer. Along the way, students can share their insights and reflections, listen to others' insights and perspectives, and brainstorm solutions. Collaboration and inquiry are thus at the heart of this learning model, and communication serves as its medium.

In this study, a context-specific, problem-oriented collaborative inquiry instructional model is constructed, taking into account the characteristics of 4th grade textbooks and collaborative inquiry-based learning. Through collaboration and inquiry, mediated by student-to-student and teacher-to-student communication, it aims to promote reasoning and analytical thinking skills among students.

4. Results

4.1. Contextual Introduction (Question 1)

Teacher introduces context relevant to the knowledge of the lesson to build a learning environment and stimulate the curiosity of students. Taking the teaching content shown in Figure 1 as an example, the teaching objectives of this lesson are to let students learn the estimation method in the process of solving practical problems, explore the calculation method of multiplying three-digit by two-digit numbers, and properly use the vertical formula to calculate the calculation of multiplying three-digit by two-digit numbers. The teacher introduces the context: "It takes 114 minutes for the first artificial Earth satellite launched by our country to go around the Earth once" and asks the question: "Can you calculate the time it takes for an artificial Earth satellite to go around the Earth 2 times, 10 times, and 20 times ; Tell us how you calculated 114 x 10 and 114 x 20 ". Purpose of the instructional design is to stimulate the interest of the students through current events topics, and naturally introduces written calculations by multiplying three digit numbers by two digit numbers. In this session, we emphasize the teacher's construction of the situation to create a context for the students based on a specific problem, while also placing a focus on fostering independent thinking among the students.

4.2. Collaborative Inquiry (Question 2)

Students form small groups to investigate a math problem collaboratively. Group members discuss, share ideas, and work together to solve problems, leading to a common group outcome. During the process of collaborative inquiry learning, teachers should observe the state of group learning and provide support to groups seeking help. Taking the instructional lesson in Figure 1 as an example, the second green dot highlights the central question: 'How much time does it take an artificial Earth satellite to travel 21 times around the Earth?' In collaborative inquiry learning, students need to think independently rather than simply accepting the results of others. As shown in the second focal point of Figure 1 there are four possible ways to solve this problem. Through collaborative inquiry-based learning in small groups, members were able to comprehend these four solutions and more.

4.3. Communication /Sharing (First Round)

After the first round of collaborative inquiry-based learning, the groups developed specific learning outcomes. During this session, the learning results will not only be shared within groups but also exchanged and discussed between groups. As facilitators of learning activities, teachers will organize reporting and in-depth exchanges of group outcomes. For example, after one group has shared its method for calculating 114 x 21, different groups can add their own methods, provide comments, or initiate a discussion about the most concise method. Such generative outcomes are commonly the most rewarding part of instruction.

4.4. Collaborative Inquiry (Question 3)

In the BNU primary mathematics textbook, each lesson consists of three questions, each of which demonstrates the depth of knowledge and provides additional insights. In the collaborative inquiry-based instructional model, a second round of collaborative inquiry is designed to encourage deeper levels of thought. As shown in the third green dot in Figure 1 please calculate '135×74' using the vertical multiplication method and explain the considerations during this process. The central objective of this lesson is to instruct students in using the vertical multiplication method and guide them in exploring the rules of multiplication, as students often make mistakes with multiplication formulas.

4.5. Communication /Sharing (Second Round)

The design principles for the second round of communication and sharing are the same as for the first round, so no additional details will be given. To give an example, when a group gives advice on how to compute 135 x 74 by vertical calculation, a standardized rule for multiplication vertical calculation is eventually developed across the class through communication and sharing between groups.

4.6. Review /Conclusion

Group members reflect and summarize what they learned in the lesson. The teacher guides the groups to representation and communication with each other. For example, groups reflect and summarize how to multiply three digits by two digits and what specific rules of computation they need to follow when using a vertical computation to multiply three digits by two digits.

Figure 2 illustrates the steps of collaborative inquiry-based learning.

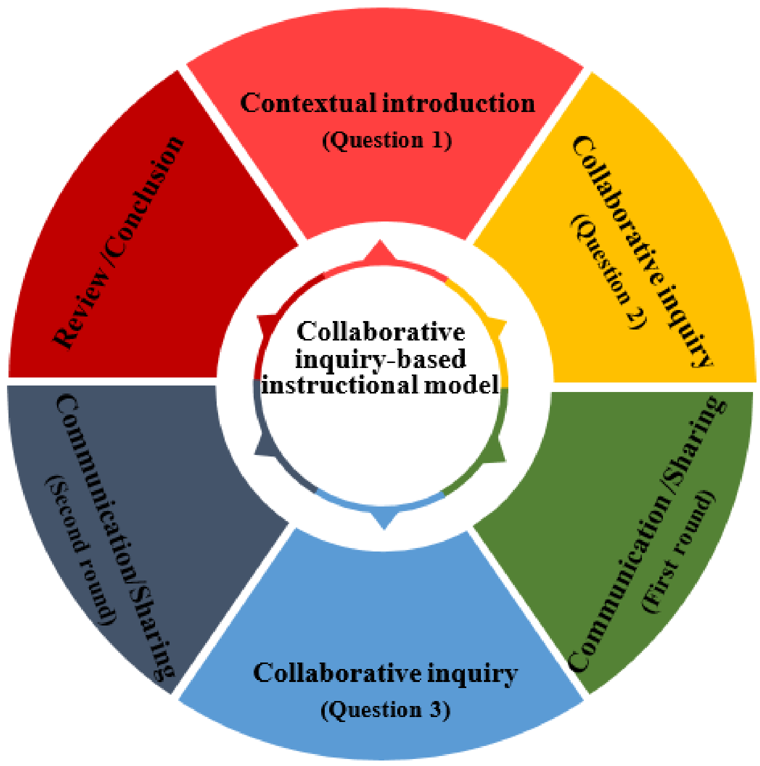

Figure 2. Collaborative inquiry-based instructional model.

5. Discussion

5.1. Philosophy of Model Design

The collaborative inquiry-based instructional model designed in present study integrates the basic elements of collaborative inquiry learning with the characteristics of primary mathematics textbooks. The instructional model consists of six main components: Contextual Introduction (Question 1), Collaborative Inquiry (Question 2), Communication and Sharing (First round), Collaborative Inquiry (Question 3), Communication and Sharing (Second round), Review and Conclusion. The next section describes the design philosophy of each part of the model.

5.1.1. Contextual Introduction

Contextualized learning is a pedagogical approach that aims to closely integrate the learning environment with real-life situations in order to enhance the practical application and relevance of learning (Hwang, Hariyanti, Chen, & Purba, 2023; Johnson, 2002). This pedagogical approach emphasizes the placement of knowledge and skills in specific contexts or situations to enable students to better understand and apply what they have learned. Grade 4 (9-10 years old) is in the concrete operations stage, and although students have begun to develop their ability to think abstractly during the concrete operations stage, their abstract thinking is still limited. They may have difficulty understanding abstract concepts and symbols and need to rely on specific situations and examples to help them understand (Piaget, 1983). Therefore, the present study introduces contextualization to the collaborative inquiry-based instructional model. It stimulates the student's interest in the topic by introducing the learning context and questions. Provide a real-world scenario in which students are motivated to learn through questions.

5.1.2. Collaborative Inquiry

Vygotsky emphasized that learning takes place primarily in an environment of social interaction and co-operation, and that interaction between learners and their peers or teachers is the primary means of constructing knowledge (Vygotsky, 1986). The zone of proximal development (ZPD) is a fundamental principle of Vygotsky's theory, indicating the gap between students' current level of development and their future level of development (Vygotsky, 1978). This definition highlights the notion that individuals undergo a process of dual development. The tangible level of development concerns the cognitive abilities that the individual has effectively acquired and can employ autonomously. In contrast, latent developmental levels encompass mental functions that individuals have not yet achieved independently but can enhance through collaboration with more proficient peers. Thus, the purpose of collaborative inquiry-based learning is to facilitate students in bridging this gap and advancing their latent development by working alongside more proficient peers.

5.1.3. Communication and Sharing

Vygotsky focuses on the impact of social interaction, language, and culture on learning (Fosnot & Perry, 2005; Jonassen, Davidson, Collins, Campbell, & Haag, 1995; Vrasidas, 2000). He emphasized the development of cognitive functions stems from social interactions, and dialogue is key to cognitive processes (Fernyhough, 2008; Mcleod, 2022; Teo, 2019). He argues that learning is not only about the internal thought processes of individuals, but also involves interactions with adults and peers in dialogue, questioning, interpretation and negotiation of meaning (Fosnot, 1996). Vygotsky considers meaning generation as a process of sharing different perspectives and experiences within a community, and that learning emerges from abundant conversations that can take place with others who have similar or different perspectives (Jonassen et al., 1995; Jonassen, 1999).

5.1.4. Review and Conclusion

Meta-cognition is the awareness of one's own cognitive processes, i.e. the ability to monitor and adjust one's own learning processes (Efklides, 2006; Padmanabha, 2020). The review and conclusion provide an opportunity for students to self-assess, helping them to reflect on the learning process and recognize their own levels of understanding and knowledge so that they can better adapt their learning strategies (Hung, 2019; Mannion, 2022).

5.2. Recommendations for Implementation

The model is designed on the basis of the essential elements of collaborative inquiry-based learning and the characteristics of the primary mathematics textbook from Beijing Normal University, and is therefore specific. When using this collaborative inquiry-based instructional model, educators in different countries should adapt it to the characteristics of their own disciplines and teaching materials.

Not all instructional content requires the use of collaborative inquiry-based teaching. On one hand, the perspectives of Kirschner, Sweller, and Clark emphasize the limitations of working memory and the advantages of direct teaching methods in reducing the burden on working memory (Kirschner, Sweller, & Clark, 2006). On the other hand, the views of Hmelo-Silver, Duncan, and Chinn highlight the support and assistance provided in constructivist learning environments to help students manage the burden on working memory (Hmelo-Silver, Duncan, & Chinn, 2007). An analysis conducted by Chinn, Barzilai, and Duncan (2020) and Chinn, Barzilai, and Duncan (2021) raises questions about the potential of direct teaching in fostering students' engagement in real-world inquiry skills. Chinn stressed the need for students to practice and explore in complex real-world settings to develop their reasoning abilities in the digital media age. Direct teaching may not provide adequate opportunities for students to confront this complexity and challenge and learn how to adapt and respond. Instead, by involving students in inquiry within real-world tasks and contexts, they can better understand and address these tangled pieces of information. It is important to note, however, that this does not mean that direct instructions are not valuable in other domains or specific contexts. Different learning tasks and disciplines may necessitate a variety of teaching methods and strategies. Some learning tasks may be better suited for direct teaching, while others may be better suited for collaborative inquiry-based learning.

5.3. Limitations and Prospects

This instructional model has been designed solely based on current research and theories to theoretically analyze its potential positive effects on improving the analytical thinking and reasoning skills of students in the primary school mathematics curriculum. However, as it has not been validated through practice, a follow-up study is planned to conduct a quasi-experiment. The study will involve selecting 4th-grade students as experimental participants and dividing them into an experimental group, which will use the collaborative inquiry-based instructional model, and a control group, which will follow a traditional teaching model. After a a period of educational intervention period, changes in students' analytical thinking and reasoning skills will be compared between the two groups, using specific assessment tools and methods

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