International Journal of Modern Research in Electrical and Electronic Engineering
Vol. 10, No. 1, 54-61, 2026
ISSN(E): 2521-0181
DOI: 10.20448/ijmreer.v10i1.8863
© 2026 by the authors; licensee Asian Online Journal Publishing Group

Control of robot motion in radial mass density field

Branko Novakovic[1] ✉
Dubravko Majetic[2]
Josip Kasac[3]
Danko Brezak[4]

(✉ *Corresponding Author)*

[1,2,3,4]*Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Ivana Lucica 5. Zagreb, Croatia.*
[1]*Email: branko.novakovic@fsb.hr*
[2]*Email: dubravko.majetic@fsb.hr*
[3]*Email: josip.kasac@fsb.hr*
[4]*Email: danko.brezak@fsb.hr*

Abstract

The proposed method models robot motion between maximal and minimal radial mass density values corresponding to minimal and maximal gravitational radii. The nonlinear dynamics of robot motion are transformed into an equivalent linear control problem using the concept of external linearization. The approach introduces a variable step parameter to regulate motion precision in the radial direction. The method is further extended to electromagnetic and gravitational multi-potential fields and illustrated through analytical examples. The results show that maximal radial mass density occurs at the minimal gravitational radius, while minimal radial mass density appears at the maximal gravitational radius. The introduced variable-step control strategy enables precise regulation of robot motion trajectories. The study also derives the energy conservation constant from the ratio of Planck mass and Planck radius and demonstrates its relation to radial mass density behavior. The proposed RRDT-based framework provides a generalized method for controlling robot motion in radial mass density fields. The use of external linearization and variable-step control enables stable and accurate robot motion in electromagnetic and gravitational environments. The proposed control strategy can be applied to nanorobotic systems, particularly in applications requiring highly precise motion control, such as medical microrobots and drug delivery systems.

Keywords: Bio/soft robots, Chemical actuated robots, Electrical robots, Energy conservation constant, Macro (Micro, nano) robot control, Magnetic robots, Maximal (Minimal) radial mass density, Radial mass density field, Robot motion control.

Citation | Novakovic, B., Majetic, D., Kasac, J., & Brezak, D. (2026). Control of robot motion in radial mass density field. *International Journal of Modern Research in Electrical and Electronic Engineering*, *10*(1), 54–61. 10.20448/ijmreer.v10i1.8863
History:
Received: 7 March 2026
Revised: 20 May 2026
Accepted: 17 June 2026
Published: 24 June 2026
Licensed: This work is licensed under a Creative Commons Attribution 4.0 License
Publisher: Asian Online Journal Publishing Group

Funding: This study received no specific financial support.
Institutional Review Board Statement: Not applicable.
Transparency: The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.
Competing Interests: The authors declare that they have no competing interests.
Authors' Contributions: All authors contributed equally to the conception and design of the study. All authors have read and agreed to the published version of the manuscript.

Contents

© 2026 by the authors; licensee Asian Online Journal Publishing Group

Contribution of this paper to the literature
This study introduces a novel RRDT-based framework for robot motion control in radial mass density fields. Unlike existing approaches, it combines external linearization with variable-step radial control in combined electromagnetic and gravitational fields, enabling precise trajectory regulation for nanorobotic applications.

1. Introduction

Generally, the very large structures of robots have many application areas, such as in precise production processes, medicine for cell manipulation, drug delivery, medical image acquisition, and non-invasive intervention. For these applications, electrical, magnetic, chemical-actuated robots, and bio/soft robots can be used [1, 2]. Genetic algorithms and unsupervised machine learning are used for predicting robotic manipulation failures in force-sensitive tasks, as discussed in [3]. An integrated design and fabrication strategy for entirely soft, autonomous robots is presented in [4]. Versatile soft grippers with intrinsic electroadhesion based on multifunctional polymer actuators are pointed out in [5]. Magnetic actuation methods in bio/soft robotics are discussed in [6]. An efficient constant-time addressing scheme for parallel-controlled assembly of stress-engineered MEMS micro-robots is presented in [7].

In this article, the control of the robot's motion is described in the radial mass density field. This field is in the region from the minimal radius (with the maximal radial mass density ($\rho_{r\ max}$)) and the maximal radius (with the minimal radial mass density ($\rho_{r\ min}$)). Between these two limited values, one can choose n points ($n=1,2,..n_{max}$). In the case of precise robot motion, the number n_{max} should be larger. Conversely, for less precise robot motion, the number n_{max} may be smaller. In that sense, one can introduce the related steps number (n_{step}) between maximal and minimal radii in a gravitational field. This value can be calculated using the relation ($n_{step} = n_{max} / n_{var}$). If a smaller parameter (n_{var}) is used than the number of steps (n_{step}), the latter is bigger, and vice versa. In that way, the most precise control of the robot's motion can be obtained.

The very important consequence of solving the field equations by including the gravitational energy-momentum tensor (EMT) on the right side of the field equation is that the gravitational field exhibits both repulsive (positive) and attractive (negative) gravitational forces. The minimum time transition between quantum states in the gravitational field is discussed in [8]. To precisely follow the desired trajectory of robot motion, one can include the new Relativistic Radial Density Theory (RRDT). The particle transition and correlation in quantum mechanics are examined in [9]. Independent position control of two identical magnetic micro-robots in a plane using permanent magnets and magnetically powerful microrobots is presented in [10]. This application signifies a new approach to the medical revolution epoch. Magnetically powered micro-robots are discussed in [11, 12].

Further, the robust control of micro-robot motion is presented in [13]. A conjugate gradient-based BPTT–like optimal control algorithm with vehicle dynamics control application is discussed. Robust motion control with an anti-windup scheme for electromagnetic actuated microrobots using time-delay estimation is presented in [14]. The two independent position controls of two microrobots moving in a plane are realized using rotating permanent magnets [10]. Magnetically powered microrobots and robust motion control, with an anti-windup scheme for electromagnetic actuated microrobots, are presented in Chautems, et al. [11] and Kim, et al. [14] respectively. Robotic-assisted minimally invasive surgery is illustrated in [15]. The design of a novel haptic joystick for the teleoperation of continuum-mechanism-based medical robots is presented in [16]. In this reference, a novel mechanism with a series of coupled gears, aimed at controlling continuum robots for medical applications, is highlighted. Positioning control of robotic manipulators subject to excitation from non-ideal sources is discussed in [17]. Further, tractor-robot cooperation is illustrated in [18]. Indoor positioning systems of mobile robots are presented in [19]. A new single-leg lower-limb rehabilitation robot motion is presented in [20]. Multi-robot task scheduling for consensus-based fault-resilient intelligent behavior in smart factories is discussed in [21]. A new single-leg lower limb rehabilitation robot with design, analysis, and experimental evolution is presented in [20]. It is also important to know how the portable surveillance robots can be used in IoT applications [22]. The recent trends in robot learning and evolution for swarm robotics are presented in [23]. Finally, the proactivity of fish and leadership of self-propelled robotic fish during interaction and bioinspiration with biomimetics is discussed [24].

2. Dynamics of Autonomous Robot Motion in Two-Potential Electromagnetic and Gravitational Radial Mass Density Field

The problem of nonlinear control of robot motion is discussed as a function of the maximal radial mass density value. To simplify the related calculations, it starts with the concept of external linearization of the nonlinear control of robot motion in the radial mass density field. In that case, within the closed regulation loop, the system exhibits linear behavior. Therefore, the problem of robot position control in the radial mass density field can be approached by calculating the control of the error vector, *e(t)*. This vector is a function of the radial mass density, ρ_r, and can be presented by the relations:

$$e = X_w - X,\quad \frac{d^2e}{dt^2} = r_w(t) - \frac{n}{\rho_{r\,max} r_{min}}\left[F_p + F_t + \frac{1}{c} N F_I\right],$$
$$r_w(t) = \frac{d^2 X_w}{dt^2} = \frac{1/n}{\rho_{r\,max} r_{min}}\left[F_{pw} + F_{tw} + \frac{1}{c} N F_{Iw}\right]. \tag{1}$$

Here $n=1,2,...,n_{max}$ and $n_{max} = \rho_{r\,max} / \rho_{r\,min}$, while the subscript w denotes the desired robot motion. The variables without this subscript present the real autonomous robot motion. Further, F_p is a potential force, F_t is a time-variation force, F_i is the interaction force, and N is the related connection parameter. At the same time, the relations (1) also describe the canonical differential equations of the robot motion in the combination of

© 2026 by the authors; licensee Asian Online Journal Publishing Group

electromagnetic and gravitational fields. Vector $r_w(t)$ presents the desired (nominal) acceleration of the robot motion in the radial mass density field.

Now following the idea of external linearization, one can introduce the following substitution.

$$u(t)=\frac{d^2e}{dt^2}=r_w(t)-\frac{n}{\rho_{r\,max}r_{min}}\left[F_p+F_t+\frac{1}{c}NF_I\right],\qquad u(t)=(u_x(t)\,u_y(t)\,u_z(t))^T. \quad (2)$$

Here $u(t)$ is the internal control vector of the robot motion in the radial mass density field. Further, one can apply the state-space phase variables, $(z_1\ z_2\ z_3)^T$, that from (1) gives the related state-space model of the robot motion in the radial mass density field.

$$e=(e_x\,e_y\,e_z)^T=Z_I=(z_1\,z_2\,z_3)^T,\ \frac{de}{dt}=(\frac{de_x}{dt}\,\frac{de_y}{dt}\,\frac{de_z}{dt})^T=Z_{II}=(z_4\,z_5\,z_6)^T, \quad (3)$$

and

$$dZ/dt=AZ(t)+Bu(t),\ \ A=\begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix},B=\begin{bmatrix}0\\ I\end{bmatrix},\quad I=diag\left[1,1,1\right]. \quad (4)$$

In (4), parameters A and B are constant matrices with dimensions (6x6) and (6x3), respectively. It is assumed that the disturbances in a state-space model of the robot's motion in the radial mass density field (3) and (4) are of the initial condition types. To eliminate the control error caused by disturbances in the radial mass density field, one can introduce the following internal control:

$$F_p=\frac{1}{n}\rho_{r\,max}r_{min}\left[r_w(t)+K_IZ_I+K_{II}Z_{II}\right]-\left[F_t+\frac{N}{c}F_I\right],\qquad u(t)=-\,KZ\ . \quad (5)$$

Here, K is the state space controller, Z is the control error, F_p is the potential force, F_t is the time-variable force, F_I is an interaction force, N is a constant, and c is the speed of light in vacuum. Including the internal control relations (3) and (4) into (5), one obtains the related equation of the potential force as a function of the radial mass density value in the linear form:

$$F_p=\frac{1}{n}\rho_{r\,max}r_{min}\left[r_w(t)+K_IZ_I+K_{II}Z_{II}\right]-\left[F_t+\frac{N}{c}F_I\right]. \quad (6)$$

Now starting from the previous relations, one can generate the new equations of the potential force F_p as functions of the potential energies U_j and U_c.

$$F_{p_x}=-\left(\frac{\partial\Sigma U_j}{\partial x}+\frac{\partial U_c}{\partial x}\right),\ \ F_{p_y}=-\left(\frac{\partial\Sigma U_j}{\partial y}+\frac{\partial U_c}{\partial y}\right),\qquad F_{p_z}=-\left(\frac{\partial\Sigma U_j}{\partial z}+\frac{\partial U_c}{\partial z}\right). \quad (7)$$

Here, $j = g$ for the gravitational field, $j = e$ for the electromagnetic field, and U_c is the related control potential field. It is followed by the inclusion of the control potential force, F_{cp}, derived from the artificial control field with potential control energy U_c. After including relation (7) into relation (6), one obtains the nonlinear control of robot motion in the multi-potential field as a function of the maximal radial mass density $\rho_{r\,max}$:

$$F_{cp}=\frac{1}{n}\rho_{r\,max}r_{min}\left[r(t)+K_IZ_I+K_{II}Z_{II}\right]-\left[F_{dp}+F_t+\frac{N}{c}F_I\right]. \quad (8)$$

Now, using (8), the control of the nonlinear system is solved by employing the concept of external linearization in the radial mass density field. Here, the obtained equations are functions of the radial mass density values.

The general approach to controlling the dynamics of robot motion in a radial mass density field for more potential fields, given in (8), can also be applied to the two potential electromagnetic and gravitational fields. In this sense, let a robot be an electrically charged particle with charge q and rest mass m_0 that moves at a non-relativistic velocity ($v << c$) in combined electromagnetic and gravitational potential fields. Further, it is also assumed that the gravitational field is produced by a spherically symmetric (non-charged) body with mass M. In that case, the total potential energy U of the robot motion in the two potential radial mass density fields is described by the relation:

$$U=qV_e+m_0V_g=qV_e+m_0\left(-\frac{GM}{r}\right),\qquad \rho_{r\,max}=\frac{m_0}{r_{min}},\ U=qV_e+\frac{1}{n}\rho_{r\,max}r_{min}\left(-\frac{GM}{r}\right). \quad (9)$$

Here, V_e and V_g are the related scalar potentials of the electromagnetic and gravitational radial mass density fields, respectively. Parameter G is the gravitational constant, and r is the radius, representing the distance between the autonomous robot and the center of mass M. Also, $n=1,2,..,n_{max}$, $n_{max}=\rho_{r\,max}/\rho_{r\,min}$. Now, applying (1) and using the notations (E_e,H_e) for an electromagnetic field and (E_g, H_g) for the gravitational field, one can generate the vector equation as explicit functions of the Lorentz forces.

$$\frac{1}{n}\rho_{r\,max}r_{min}\frac{d^2X}{dt^2}=q\left(E_e+\frac{1}{c}v\times H_e\right)+\frac{1}{n}\rho_{r\,max}r_{min}\left(E_g+\frac{1}{c}v\times H_g\right). \quad (10)$$

The parameters E_e, E_g, H_e and H_g are vectors described by the relations.

$$E_e=\begin{bmatrix}E_{e_x}\\ E_{e_y}\\ E_{e_z}\end{bmatrix},\ \ E_g=\begin{bmatrix}E_{g_x}\\ E_{g_y}\\ E_{g_z}\end{bmatrix},\ \ H_e=\begin{bmatrix}H_{e_x}\\ H_{e_y}\\ H_{e_z}\end{bmatrix},\ \ H_g=\begin{bmatrix}H_{g_x}\\ H_{g_y}\\ H_{g_z}\end{bmatrix}. \quad (11)$$

In this example, a robot is a particle with charge q and rest mass m_0, and therefore, this robot interacts with both electromagnetic and gravitational radial mass density fields. In that sense, the relations (10) and (11) describe the dynamics of the robot's motion in two potential fields: electromagnetic and gravitational. The components of the vector E_e and E_g can be calculated using the following equations.

© 2026 by the authors; licensee Asian Online Journal Publishing Group

$$E_{e_x} = -\frac{\partial V_e}{\partial x} - \frac{1}{c}\frac{\partial A_{e_x}}{\partial t}, \quad E_{g_x} = -\frac{\partial V_g}{\partial x} - \frac{1}{c}\frac{\partial A_{g_x}}{\partial t}, \quad E_{e_y} = -\frac{\partial V_e}{\partial y} - \frac{1}{c}\frac{\partial A_{e_y}}{\partial t}, \quad (12)$$

and

$$E_{g_y} = -\frac{\partial V_g}{\partial y} - \frac{1}{c}\frac{\partial A_{g_y}}{\partial t}, \quad E_{e_z} = -\frac{\partial V_e}{\partial z} - \frac{1}{c}\frac{\partial A_{e_z}}{\partial t}, \quad E_{g_z} = -\frac{\partial V_g}{\partial z} - \frac{1}{c}\frac{\partial A_{g_z}}{\partial t}. \quad (13)$$

The components of vectors A_e, A_g, H_e, and H_g in (12) and (13) are given by the relations.

$$A_{e_i} = \left(\frac{v_i V_e}{c}\right), \quad A_{g_i} = \left(\frac{v_i V_g}{c}\right), \quad H_{e_x} = \frac{\partial A_{e_z}}{\partial y} - \frac{\partial A_{e_y}}{\partial z}, \quad H_{g_x} = \frac{\partial A_{g_z}}{\partial y} - \frac{\partial A_{g_y}}{\partial z}, \quad i = x,y,z, \quad (14)$$

and

$$H_{e_y} = \frac{\partial A_{e_x}}{\partial z} - \frac{\partial A_{e_z}}{\partial x}, \quad H_{g_y} = \frac{\partial A_{g_x}}{\partial z} - \frac{\partial A_{g_z}}{\partial x}, \quad H_{e_z} = \frac{\partial A_{e_y}}{\partial x} - \frac{\partial A_{e_x}}{\partial y}, \quad H_{g_z} = \frac{\partial A_{g_y}}{\partial x} - \frac{\partial A_{g_x}}{\partial y}. \quad (15)$$

Applying (6) and (7) to the canonical differential equations of the autonomous robot motion in the two-potential radial mass density field results in the control error model of the robot's motion as a function of the maximal radial mass density value:

$$\ddot{e}(t) = r_w(t) - \frac{nq}{\rho_{r\,max} r_{min}}\left(E_e + \frac{1}{c}\mathrm{v}\times H_e\right) - \left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right), \quad (16)$$

and

$$r_w(t) = \frac{nq}{\rho_{r\,max} r_{min}}\left(E_{e_w} + \frac{1}{c}\mathrm{v}_w\times H_{e_w}\right) - \left(E_{g_w} + \frac{1}{c}\mathrm{v}_w\times H_{g_w}\right). \quad (17)$$

In (17), r_w is the vector of desired acceleration of the robot motion. The subscript *w* denotes the desired values of the related variables. The next step is the application of the *concept* of external linearization to transform Equation 16 into the new relation.

$$u(t) = r_w(t) - \frac{nq}{\rho_{r\,max} r_{min}}\left(E_e + \frac{1}{c}\mathrm{v}\times H_e\right) - \left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right). \quad (18)$$

Here, $u(t)$ is the internal control vector, and $n=1,2,..,n_{max}$ is the number of robot steps from the minimal to the maximal radii in the radial mass density field. From (17) and (18), one obtains the related equivalent of the linear control error model of robot motion in the combined electromagnetic and gravitational radial mass density field, given by (6) and (7). Applying (18), one obtains the new relation as a function of the maximal radial mass density in this form.

$$E_e = \frac{\rho_{r\,max} r_{min}}{nq}\left[r_w(t) + K_I Z_I + K_{II} Z_{II}\right] - \left(\frac{1}{c}\mathrm{v}\times H_e\right) - \frac{\rho_{r\,max} r_{min}}{nq}\left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right). \quad (19)$$

Now, let the electric field E_e consist of two electric components: $E_e = E_{de} + E_{ce}$. Here, E_{de} is a disturbance electric field caused by the influence of a two-potential field on the motion of the robot in the radial mass density field. The component E_{ce} is an artificial electric control field that should control the robot motion in the two potential fields. Including $E_e = E_{de} + E_{ce}$ into (19), one obtains the nonlinear electric control of the robot motion in the two-potential radial mass density field as a function of the maximal radial mass density.

$$E_{ce} = \frac{\rho_{r\,max} r_{min}}{nq}\left[r_w(t) + K_I Z_I + K_{II} Z_{II}\right] - \left(E_{de} + \frac{1}{c}\mathrm{v}\times H_e\right) - \frac{\rho_{r\,max} r_{min}}{nq}\left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right). \quad (20)$$

Taking into account the relation (10), the canonical differential equations of the robot motion in the two-potential radial mass density field can be rewritten as a function of the maximal radial mass density:

$$\frac{d^2X}{dt^2} = \frac{nq}{\rho_{r\,max} r_{min}}\left(E_{de} + E_{ce} + \frac{1}{c}\mathrm{v}\times H_e\right) + \left(E_g + \frac{1}{c}\mathrm{v}\times H_g\right). \quad (21)$$

Applying the nonlinear control E_{ce} from (20) to the nonlinear dynamical model of the robot motion (21), one obtains the closed-loop system in linear form:

$$\frac{d^2X}{dt^2} = r_w(t) + K_I Z_I + K_{II} Z_{II}. \quad (22)$$

Thus, Equation 20 is the nonlinear control, which in the closed loop with the nonlinear canonical differential equations of the robot motion (21), results in the linear behavior of the whole system (22). In that way, the problem of controlling the robot motion in the combination of an electromagnetic and gravitational radial mass density field has been solved by employing the concept of external linearization. This is very important for the application of micro and nanorobots in drug delivery across the human body.

3. The Other Methods of Application of the Radial Mass Density to Robot Control

The global positioning of robot manipulators with mixed revolute and prismatic joints is presented in [14]. This section illustrates how to apply the maximal radial mass density theory to this class of robots. The dynamic model of the n-link rigid body robot can be described as a function of the maximal radial mass density:

$$m_0(q)\frac{d^2q}{dt^2} + C(q,\frac{dq}{dt})\frac{dq}{dt} + q(q) = U, \quad m_0(q) = \rho_{r\,max} r_{min}.$$
$$\rho_{r\,max} r_{min}(q)\frac{d^2q}{dt^2} + C(q,\frac{dq}{dt})\frac{dq}{dt} + q(q) = U, \quad (23)$$

© 2026 by the authors; licensee Asian Online Journal Publishing Group

Here, q is a $(nx1)$ vector of robot joint coordinates, dq/dt is the related vector of joint velocities, U is a vector of applied joint torques and forces, $m_0(q)$ is (nxn) inertia matrix, and $C(q, dq/dt)dq/dt$ is an $(nx1)$ vector of centrifugal and Coriolis torques. Further, $q(q)$ is the vector of gravitational torques and forces, and $\rho_{r\,max}$ is the maximal radial mass density at the minimal radius. If the robot, described by (23), is in the closed loop with the nonlinear *PID* controller, described by the relation.

$$U(t) = -(K_p \frac{d^2q}{dt^2} + K_d \frac{dq}{dt} + K_I \frac{dq}{dt}). \qquad (24)$$

Then the closed-loop system of the relations (23) and (24) resulted in the form that is the function of the maximal radial mass density.

$$\rho_{r\,max} r_{min}(q)\frac{d^2q}{dt^2} + C(q,\frac{dq}{dt})\frac{dq}{dt} + q(q) = -(K_p \frac{d^2q}{dt^2} + K_d \frac{dq}{dt} + K_I \frac{dq}{dt}). \qquad (25)$$

The relation (25) can be applied for the parameter $n = 1,2,\ldots,\rho_{r\,max} / \rho_{r\,min}$. Now, one can use the relation (25) in the new form.

$$\frac{d^2q}{dt^2} = -\frac{n_{step}}{\rho_{r\,max} r_{min}(q)}(C(q,\frac{dq}{dt})\frac{dq}{dt} + q(q) + K_p \frac{d^2q}{dt^2} + K_d \frac{dq}{dt} + K_I \frac{dq}{dt}),\ n_{step} = n_{max} / n_{var}. \qquad (26)$$

Thus, using the relation (26), it is possible to control the robot's acceleration by changing the numerical parameter n_{step}. In that way, by changing the parameter n_{var}, it is possible to realize the most precise robot motion control. This means that the radial distance between two points should be minimal if the n_{var} is maximal.

The dynamics of robot motion can also be described as a function of the alpha field parameters derived in the Relativistic Alpha Field Theory (*RAFT*) [7]. In this theory, one can start with the potential energy of the robot (particle) in the combination of electromagnetic and gravitational fields, U_e and U_g, respectively. Now let q, m, V_e, and V_g be the robot's (particle's) charge, mass, electrical potential, and gravitational potential, respectively. Further, G is the gravitational constant, M is the mass of the gravitational field, and c is the speed of light in a vacuum. The potential energy of the robot in the combination of electromagnetic and gravitational fields is given by the relations:

$$U = U_e + U_g = \pm qV_e - \frac{mGM}{r},\ \frac{U}{mc^2} = \pm\frac{qV_e}{mc^2} - \frac{GM}{rc^2}, \qquad (27)$$

and

$$m = \rho_{r\,max} r_{min},\ \frac{U}{\rho_{r\,max} r_{min} c^2} = \pm\frac{qV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}. \qquad (28)$$

The relation (28) can also be described as the function of the parameter n:

$$\frac{nU}{\rho_{r\,max} r_{min} c^2} = \pm\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2},\ n = 1\ldots,n_{max},\ n_{step} = n_{max} / n_{var}. \qquad (29)$$

If one wants to use the *RAF* theory in robotics, then it requires the introduction of the related alpha field parameters. The solution of the field parameters for an electron in the two-potential electromagnetic and gravitational fields is given as follows. In that sense, parameters α_1 and α'_1 are given by the relations.

$$\alpha_1 = 1 + i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}},\ \alpha'_1 = 1 - i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}. \qquad (30)$$

For parameters α_2 and α'_2.

$$\alpha_2 = 1 - i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}},\ \alpha'_2 = 1 + i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}. \qquad (31)$$

For parameters α_3 and α'_3.

$$\alpha_3 = -1 + i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}},\ \alpha'_3 = -1 - i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}, \qquad (32)$$

and for parameters α_4 and α'_4.

$$\alpha_4 = -1 - i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}},\ \alpha'_4 = -1 + i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}. \qquad (33)$$

Now one can introduce the generalized Lorentz-Einstein parameters for an electron in a two-potential electromagnetic and gravitational field. These parameters are described by the following equations:

© 2026 by the authors; licensee Asian Online Journal Publishing Group

$$H_{1,2} = \left[1 - \frac{v^2}{c^2 + \frac{nqV_e}{\rho_{r\,max} r_{min}} - \frac{GM}{r}} \pm \frac{2i\sqrt{\frac{nqV_e}{\rho_{r\,max} r_{min} c^2} - \frac{GM}{rc^2}}\; c \cdot v}{c^2 + \frac{nqV_e}{\rho_{r\,max} r_{min}} - \frac{GM}{r}} \right]^{-1/2} . \quad (34)$$

Thus, the solutions of the $H_{3,4}$ are symmetric to the solutions of the parameters H_{12}.

The previously presented two potential fields can be generalized by applying the multi-potential field as a function of the field parameters α and α'. To derive a four-potential vector A of the related potential field, recall the general Hamilton function, H, for weak potential fields:

$$H = -c\zeta_1\left(p_x - \frac{U_p v_x}{c^2}\right) - c\zeta_2\left(p_y - \frac{U_p v_y}{c^2}\right) - c\zeta_3\left(p_z - \frac{U_p v_z}{c^2}\right) - \frac{\beta \rho_{r\,max} r_{min}}{n} c^2 + U_p , \quad (35)$$

$$n = 1 \ldots, n_{max}, \quad n_{step} = n_{max} / n_{var} .$$

Here, U_p is potential energy, (p_x, p_y, p_z) is a three-momentum vector, (v_x, v_y, v_z) is a three-velocity vector, and ζ_1, ζ_2, ζ_3, and β are the well-known Dirac matrices. If an electron moves with a constant velocity v << c in an electromagnetic field with scalar potential, V, then one should use the following relations.

$$\frac{U_p v_x}{c^2} = \frac{q}{c}\frac{V v_x}{c} = \frac{q}{c} A_x , \quad \frac{U_p v_y}{c^2} = \frac{q}{c}\frac{V v_y}{c} = \frac{q}{c} A_y , \quad \frac{U_p v_z}{c^2} = \frac{q}{c}\frac{V v_z}{c} = \frac{q}{c} A_z . \quad (36)$$

Here, q is the electric charge of an electron, and (A_x, A_y, A_z) is the three-potential vector of the electromagnetic field. Including (36) into (35), one obtains the well-known Hamilton function for Dirac's electron in an electromagnetic field.

$$H = -c\zeta_1\left(P_x - \frac{q}{c}A_x\right) - c\zeta_2\left(P_y - \frac{q}{c}A_y\right) - c\zeta_3\left(P_z - \frac{q}{c}A_z\right) - \frac{\beta \rho_{r\,max} r_{min}}{n} c^2 + qV . \quad (37)$$

On the other hand, if a robot (particle) is moving with constant velocity v << c in a gravitational field, then, according to the previous procedure, one should use the following relations.

$$U_p = -\frac{\rho_{r\,max} r_{min} GM}{nr} = \frac{\rho_{r\,max} r_{min}}{n} V_g , \quad \frac{U_p v_x}{c^2} = \frac{\rho_{r\,max} r_{min}}{nc}\frac{V_g v_x}{c} = \frac{\rho_{r\,max} r_{min}}{nc} A_{g_x} . \quad (38)$$

and

$$\frac{U_p v_y}{c^2} = \frac{\rho_{r\,max} r_{min}}{nc}\frac{V_g v_y}{c} = \frac{\rho_{r\,max} r_{min}}{nc} A_{g_y} , \quad \frac{U_p v_z}{c^2} = \frac{\rho_{r\,max} r_{min}}{nc}\frac{V_g v_z}{c} = \frac{\rho_{r\,max} r_{min}}{nc} A_{g_z} . \quad (39)$$

In the relations (38) and (39), G is a gravitational constant, M is a gravitational mass, V_g is a gravitational scalar potential, and (A_{gx}, A_{gy}, A_{gz}) is a three-potential vector of the gravitational field. Including (39) into the equation (37) yields the Hamilton function H_g for the particle in a gravitational field:

$$\begin{aligned} H_g &= -c\zeta_1\left(P_x - \frac{\rho_{r\,max} r_{min}}{nc} A_{g_x}\right) - c\zeta_2\left(P_y - \frac{\rho_{r\,max} r_{min}}{nc} A_{g_y}\right) \\ &-c\zeta_3\left(P_z - \frac{\rho_{r\,max} r_{min}}{nc} A_{g_z}\right) - \beta\frac{\rho_{r\,max} r_{min}}{n} c^2 + \frac{\rho_{r\,max} r_{min}}{n} V_g . \end{aligned} \quad (40)$$

Generally, if the robot velocity v in a potential field is constant, then the four-potential vector A can be derived as a function of the field parameters α and α′:

$$A = \left[A^0, A^1, A^2, A^3\right], \quad A^0 = \frac{c^2}{\eta}\left(\alpha\alpha' - 1\right), \quad A^1 = A_x = A^0 \frac{v_x}{c} , \quad (41)$$

and

$$A^1 = A_x = A^0\frac{v_x}{c} , \quad A^2 = A_y = A^0\frac{v_y}{c} , \quad A^3 = A_z = A^0\frac{v_z}{c} . \quad (42)$$

Now, the components of the field tensor F_{ij} of the potential field can be calculated using relations (41) and (42) and the well-known procedure.

$$F_{ij} = \frac{\partial A^j}{\partial x^i} - \frac{\partial A^i}{\partial x^j} , \quad i, j = 0,1,2,3, \quad X = \left[x^0, x^1, x^2, x^3\right] = \left[ct, x, y, z\right] . \quad (43)$$

As a result of this calculation, one obtains the well-known anti-symmetric tensor F_{ij} of the potential field in the following form.

$$F_{ij} = \begin{bmatrix} 0 & F_{01} & F_{02} & F_{03} \\ F_{10} & 0 & F_{12} & F_{13} \\ F_{20} & F_{21} & 0 & F_{23} \\ F_{30} & F_{31} & F_{32} & 0 \end{bmatrix} = \begin{bmatrix} 0 & F_{01} & F_{02} & F_{03} \\ -F_{01} & 0 & F_{12} & -F_{13} \\ -F_{02} & -F_{12} & 0 & F_{23} \\ -F_{03} & F_{13} & -F_{23} & 0 \end{bmatrix} . \quad (44)$$

This tensor can be employed for the derivation of the related Maxwell-like equations in a vacuum.

Following the previous consideration, one can introduce the normalized scalar potential A_m^0 of a multi-potential field in the dimension of specific potential energy:

$$A_m^0 = \Sigma\left(\eta_j A_j^0\right) = \left(\alpha\alpha'_j - 1\right)c^2 , \quad j = 1,2,..,n, \quad (45)$$

© 2026 by the authors; licensee Asian Online Journal Publishing Group

where term $\alpha\alpha'$ has to be calculated by employing the relations (30) to (33).

$$(\alpha\alpha')=\left(1+\frac{n\Sigma U_{p_j}}{\rho_{r\,max}r_{min}c^2}\right),\quad j=1,2,..,n. \qquad (46)$$

The relations (45) and (46) tell us what the normalized scalar potential A_m^0 really is.

$$A_m^0=\frac{n\Sigma U_{p_j}}{\rho_{r\,max}r_{min}},\quad j=1,2,..,n,\quad n=1...,n_{max},\quad n_{step}=n_{max}/n_{var}. \qquad (47)$$

In recent decades, a wide range of robotic systems has been created, mostly inspired by animals. Engineers have developed various robotic systems such as four-legged robots, snake robots, insect robots, and fish robots [25]. Following this, it is possible to control this class of robots using the radial mass density field theory.

4. Illustrative Example: Calculation of the Robot Motion in Radial Mass Density Field

The gravitational field with the mass M_g has the maximal and minimal gravitational radial mass densities given in [11].

$$\rho_{rm\,\max}=\frac{M_g}{r_{\min}}=\frac{(1+\kappa)c^2}{G}=2.693182\cdot10^{27}\,kg/m, \qquad (48)$$

and

$$\rho_{rm\,\min}=\frac{M_g}{r_{\max}}=\frac{(1-\kappa)c^2}{G}=0.888779\cdot10^{23}\,kg/m. \qquad (49)$$

The numerical values in (48) and (49) are constant and are valid for all amounts of the gravitational masses M_g. In relations (48) and (49), the parameter κ is the energy conservation constant calculated in Chautems, et al. [11] using Planck's mass and Planck's length,

$$L_p=\frac{2GM_p}{(1+\kappa)c^2},\quad \kappa=\frac{2GM_p}{L_pc^2}-1=0.99993392118. \qquad (50)$$

Thus, the value of κ is close to one but less than it. Using the combination of Equations 21 and (48), one obtains the canonical differential equations of the robot motion at the minimal gravitational radius with the maximal radial mass density.

$$\frac{d^2X}{dt^2}=\frac{nq}{2.693182\cdot10^{27}}\left(E_{de}+E_{ce}+\frac{1}{c}\mathrm{v}\times H_e\right)+\left(E_g+\frac{1}{c}\mathrm{v}\times H_g\right)m/kg, \qquad (51)$$

$$n_{step}=n_{max}/n_{var}.$$

By changing parameter n = 1, 2, ..., n_{max} and using parameter n_{var}, one can achieve the desired acceleration and precise control of robot motion within the control region. Additionally, combining (21) and (48) yields the canonical differential equations of robot motion at the maximal gravitational radius, with minimal radial mass density.

$$\frac{d^2X}{dt^2}=\frac{nq}{0.888779\cdot10^{23}}\left(E_{de}+E_{ce}+\frac{1}{c}\mathrm{v}\times H_e\right)+\left(E_g+\frac{1}{c}\mathrm{v}\times H_g\right)m/kg, \qquad (52)$$

$$n_{step}=n_{max}/n_{var}.$$

Now, by changing parameter n = 1, 2, .., n_{max} and variable parameter n_{var}, one can obtain the desired acceleration and precise motion of the robot control in the related region. The ratio between the maximal and minimal radial mass densities can be calculated using the relation.

$$n_{\max}=\frac{\rho_{rm\,\max}}{\rho_{rm\,\min}}=\frac{2.693182\cdot10^{27}}{0.888779\cdot10^{23}}=3.030204\cdot10^4. \qquad (53)$$

This ratio is the constant and is valued for all amounts of the gravitational masses.

Following the previous equations, one can calculate the maximal steps, n_{step}, between maximal and minimal radii in a gravitational field. For the precise motion of robots in the gravitational radial direction, one can introduce the variable step of the robot's motion, n_{var}. In that case, it is possible to select (change) the scale of the desired step of the robot's motion in the radial mass density field.

For example, let the variable step of the robot's motion in the radial direction be given by the amount n_{var} = 100. In that case, the number of robot steps n_{step} from the minimal to the maximal radii has the value.

$$n_{step}=\frac{n_{\max}}{n_{\mathrm{var}}}=\frac{303.0204\cdot10^2}{100}=303.0204\,steps. \qquad (54)$$

In this calculation, a robot needs 303 steps of motion from the minimum to the maximum radius in the radial direction. If the robot's motion is not radial, the related projection of the radial trajectory should be used for the desired robot trajectory. The next example relates to the possibility of introducing the potential energies at the minimal and maximal gravitational radii, $U_{g\,max}$ and $U_{g\,min}$. In this case, it is possible to calculate the minimal and maximal radial lengths, L_{gmin} and L_{gmax}, respectively, using the relations.

$$M_g=\rho_{rm\,\max}r_{\min},\quad L_{g\min}=\frac{2m_0G\rho_{rm\,\max}r_{\min}}{U_{g\min}}=\frac{2m_0G\rho_{rm\,\max}r_{p\min}}{(1+\kappa)c^2}, \qquad (55)$$

and

$$L_{g\max}=\frac{2m_0G\rho_{rm\,\min}r_{\max}}{U_{g\max}}=\frac{2m_0G\rho_{rm\,\min}r_{p\min}}{(1-\kappa)c^2}. \qquad (56)$$

© 2026 by the authors; licensee Asian Online Journal Publishing Group

From relations (55) and (56), one can see how potential energies in the gravitational field influence the robot's motion in that field.

5. Conclusion

This article is based on the new Relativistic Radial Density Theory (*RRDT*) that has been applied to the control of robot motion in potential fields. The robot's motion is calculated from the minimal to the maximal gravitational radii and vice versa. When the robot's motion is not in the radial direction, it is necessary to transform the radial coordinates into rectangular ones using the related projection. It is shown that the maximal radial mass density occurs at the minimal gravitational radius. Conversely, the minimal radial mass density happens at the maximal gravitational radius. Furthermore, both maximal and minimal radial mass densities can also be described as functions of the energy conservation constant κ. In this context, the related gravitational length, time, energy, and temperature can be represented as functions of the Planck length, time, energy, and temperature, respectively. Finally, it is concluded that the precise control of robot motion in combination with electromagnetic and gravitational fields can be achieved by introducing the variable step of the robot's motion. In that sense, one can introduce the variable number of steps, n_{step}, between maximal and minimal radii in the gravitational field by using the relation $n_{step} = n_{max} / n_{var}$. In that case, the smaller value of the parameter n_{var} results in a larger number of steps, n_{step}, and vice versa. Thus, the bigger n_{step} gives more precise control of the robot motion in the radial mass density field.

References

[1] N. Buniyamin, W. A. J. Wan Ngah, N. Sariff, and Z. Mohamad, "A simple local path planning algorithm for autonomous mobile robots," *International Journal of Systems Applications, Engineering & Development*, vol. 5, no. 2, pp. 151-159, 2011.

[2] L. X. Lyu *et al.*, "Bio-inspired untethered fully soft robots in liquid actuated by induced energy gradients," *National Science Review*, vol. 6, no. 5, pp. 970-981, 2019. https://doi.org/10.1093/nsr/nwz083

[3] L. Parisi and N. RaviChandran, "Genetic algorithms and unsupervised machine learning for predicting robotic manipulation failures for force-sensitive tasks," in *Proceedings of the 4th International Conference on Control, Automation and Robotics (ICCAR)*, 2018.

[4] M. Wehner *et al.*, "An integrated design and fabrication strategy for entirely soft, autonomous robots," *Nature*, vol. 536, no. 7617, pp. 451-455, 2016. https://doi.org/10.1038/nature19100

[5] J. Shintake, S. Rosset, B. Schubert, D. Floreano, and H. Shea, "Versatile soft grippers with intrinsic electroadhesion based on multifunctional polymer actuators," *Advanced Materials*, vol. 28, no. 2, pp. 231-238, 2016. https://doi.org/10.1002/adma.201504264

[6] N. Ebrahimi *et al.*, "Magnetic actuation methods in bio/soft robotics," *Advanced Functional Materials*, vol. 31, no. 11, p. 2005137, 2021. https://doi.org/10.1002/adfm.202005137

[7] V. Foroutan, F. Farzami, D. Erricolo, and I. Paprotny, "Efficient constant-time addressing scheme for parallel-controlled assembly of stress-engineered MEMS microrobots," in *2018 4th International Conference on Control, Automation and Robotics (ICCAR). IEEE*, 2018.

[8] B. Novakovic, "Minimum time transition between quantum states in gravitational field," *American Journal of Modern Physics*, vol. 10, no. 2, pp. 30-35, 2021. https://doi.org/10.11648/j.ajmp.20211002.12

[9] A. R. Ruffa, "One particle transitions and correlation in quantum mechanics," *Journal of Research of the National Bureau of Standards. Section A, Physics and Chemistry*, vol. 69A, no. 2, pp. 167–180, 1965. https://doi.org/10.6028/jres.069A.019

[10] M. Yousefi and H. Nejat Pishkenari, "Independent position control of two identical magnetic microrobots in a plane using rotating permanent magnets," *Journal of Micro-Bio Robotics*, vol. 17, no. 1, pp. 59-67, 2021. https://doi.org/10.1007/s12213-021-00143-w

[11] C. Chautems, B. Zeydan, S. Charreyron, G. Chatzipirpiridis, S. Pané, and B. J. Nelson, "Magnetically powered microrobots: A medical revolution underway?," *European Journal of Cardio-Thoracic Surgery*, vol. 51, no. 3, pp. 405-407, 2017. https://doi.org/10.1093/ejcts/ezw432

[12] A. Codutti, F. Bachmann, D. Faivre, and S. Klumpp, "Bead-based hydrodynamic simulations of rigid magnetic micropropellers," *Frontiers in Robotics and AI*, vol. 5, p. 109, 2018. https://doi.org/10.3389/frobt.2018.00109

[13] B. Novakovic, "Quantization and structure of electromagnetic and gravitational fields," *American Journal of Modern Physics*, vol. 10, no. 6, pp. 118-123, 2021. https://doi.org/10.11648/j.ajmp.20211006.11

[14] J. Kim, H. Choi, and J. Kim, "A robust motion control with antiwindup scheme for electromagnetic actuated microrobot using time-delay estimation," *IEEE/ASME Transactions on Mechatronics*, vol. 24, no. 3, pp. 1096–1106, 2019. https://doi.org/10.1109/TMECH.2019.2907145

[15] J. H. Palep, "Robotic assisted minimally invasive surgery," *Journal of Minimal Access Surgery*, vol. 5, no. 1, pp. 1-7, 2009. https://doi.org/10.4103/0972-9941.51313

[16] X. Yiping, X. Hou, and S. Wang, "Design of a novel haptic joystick for the teleoperation of continuum-mechanism-based medical robots," *Robotics*, vol. 12, no. 2, p. 52, 2023. https://doi.org/10.3390/robotics12020052

[17] M. T. Angelo *et al.*, "Positioning control of robotic manipulators subject to excitation from non-ideal sources," *Robotics*, vol. 12, no. 2, p. 51, 2023. https://doi.org/10.3390/robotics12020051

[18] E. H. C. Harik, "Tractor-robot cooperation: A heterogeneous leader-follower approach," *Robotics*, vol. 12, no. 2, p. 57, 2023. https://doi.org/10.3390/robotics12020057

[19] J. Huang, S. Junginger, H. Liu, and K. Thurow, "Indoor positioning systems of mobile robots: A review," *Robotics*, vol. 12, no. 2, p. 47, 2023. https://doi.org/10.3390/robotics12020047

[20] H. Yu *et al.*, "A new single-leg lower-limb rehabilitation robot: Design, analysis and experimental evaluation," *Machines*, vol. 11, no. 4, p. 447, 2023. https://doi.org/10.3390/machines11040447

[21] V. C. Kalempa, L. Piardi, M. Limeira, and A. S. de Oliveira, "Multi-robot task scheduling for consensus-based fault-resilient intelligent behavior in smart factories," *Machines*, vol. 11, no. 4, p. 431, 2023. https://doi.org/10.3390/machines11040431

[22] K. RajKumar, C. S. Kumar, C. Yuvashree, and M. S. Murugan, "Portable surveillance robot using IoT," *International Research Journal of Engineering and Technology*, vol. 6, no. 3, pp. 94-97, 2019.

[23] J. Kuckling, "Recent trends in robot learning and evolution for swarm robotics," *Frontiers in Robotics and AI*, vol. 10, p. 1134841, 2023. https://doi.org/10.3389/frobt.2023.1134841

[24] Z. Zhou, J. Liu, J. Pan, and J. Yu, "Proactivity of fish and leadership of self-propelled robotic fish during interaction," *Bioinspiration & Biomimetics*, vol. 18, no. 3, p. 036011, 2023. https://doi.org/10.1088/1748-3190/acce87

[25] S. Kim, C. Laschi, and B. Trimmer, "Soft robotics: A bioinspired evolution in robotics," *Trends in Biotechnology*, vol. 31, no. 5, pp. 287–294, 2013. https://doi.org/10.1016/j.tibtech.2013.03.002

Asian Online Journal Publishing Group *is not responsible or answerable for any loss, damage or liability, etc. caused in relation to/arising out of the use of the content. Any queries should be directed to the corresponding author of the article.*

© 2026 by the authors; licensee Asian Online Journal Publishing Group